

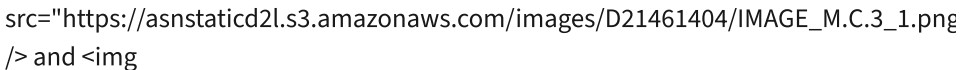

West Virginia Mathematics

# Calculus

Adopted 2024

## Calculus

### Algebra

1. Understand the key concepts, connections and applications of functions, limits, continuity, derivatives, and integrals represented in multiple ways. **C.A.1**
  1. Use abstract notation to apply properties of algebraic, trigonometric, exponential, logarithmic, and composite functions, as well as their inverses, represented graphically, numerically, analytically, and verbally; and demonstrate an understanding of the connections among these representations. **M.C.1**
  2. Demonstrate a conceptual understanding of the definition of a limit via the analysis of continuous and discontinuous functions represented using multiple representations (e.g., graphs and tables). **M.C.2**
  3. Use the properties of limits including addition, product, quotient, composition, and squeeze/sandwich theorem to calculate the various forms of limits: one-sided limits, limits at infinity, infinite limits, limits that do not exist, and special limits such as  and  **M.C.3**
  4. Apply the definition of continuity to determine where a function is continuous or discontinuous including continuity at a point, continuity over an interval, application of the Intermediate Value Theorem, and graphical interpretation of continuity and discontinuity. **M.C.4**
  5. Investigate and apply the definition of the derivative graphically, numerically, and analytically at a point; conceptually interpret the derivative as an instantaneous rate of change and the slope of the tangent line. **M.C.5**
  6. Discriminate between the average rate of change and the instantaneous rate of change using real-world problems. **M.C.6**
  7. Recognize when the Extreme Value Theorem indicates that function extrema exist. **M.C.7**
  8. Quickly recall and apply rules of differentiation including the constant multiple rule, the sum rule, the difference rule, the product rule, the quotient rule, the power rule, and the chain rule as applied to algebraic, trigonometric, exponential, logarithmic, and inverse trigonometric functions using techniques of both explicit and implicit differentiation. **M.C.8**
  9. Apply Rolle's Theorem and the Mean Value Theorem to real-world problems. **M.C.9**
  10. Construct and use mathematical models to solve optimization, related-rates, velocity and acceleration problems. **M.C.10**
  11. Determine antiderivatives that follow from derivatives of basic functions and apply substitution of variables. **M.C.11**
  12. Evaluate definite integrals using basic integration properties such as addition, subtraction, constant multipliers, the power rule, substitution, and change of limits. **M.C.12**
  13. Characterize the definite integral as the total change of a function over an interval and use this to solve real-world problems. **M.C.13**

14. Apply the Fundamental Theorem of Calculus to evaluate definite integrals and to formulate a cumulative area function and interpret the function as it relates to the integrand. **M.C.14**
  15. Use limits to deduce asymptotic behavior of the graph of a function. **M.C.15**
  16. Compare and contrast the limit definition (not delta epsilon) of continuity and the graphical interpretation of the continuity of a function at a point; recognize different types of discontinuities. **M.C.16**
  17. Develop tangent lines as best linear approximations to functions near specific points explain this conceptually; construct these tangent lines; and apply this concept to Newton's Method. **M.C.17**
  18. Investigate and explain the relationships among the graphs of a function, its derivative and its second derivative; construct the graph of a function using the first and second derivatives including extrema, points of inflection, and asymptotic behavior. **M.C.18**
  19. Approximate areas under a curve using Riemann sums by applying and comparing left, right, and midpoint methods for a finite number of subintervals. **M.C.19**
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### **Geometry**

1. Apply the key concepts, connections and applications of limits, continuity, derivatives, and integration for a wide variety of regions. **C.G.1**
  20. Justify why differentiability implies continuity and classify functional cases when continuity does not imply differentiability. **M.C.20**
  21. Calculate a definite integral using Riemann sums by evaluating an infinite limit of a sum using summation notation and rules for summation. **M.C.21**
  22. Use integration to solve problems that involve linear displacement, total distance, position, velocity, acceleration, and area between curves by looking at both functions of  $x$  and functions of  $y$ ; utilize units to interpret the physical nature of the calculus process. **M.C.22**
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### **Data Analysis and Probability**

1. Apply the key concepts and applications of limits, continuity, derivatives, and integration to analyze functions that represent a collection of data. **C.DA.1**
23. Identify a real-world situation that involves quantities that change over time; pose a question; make a hypothesis as to the answer; develop, justify, and implement a method to collect, organize, and analyze related data; extend the nature of collected, discrete data to that of a continuous function that describes the known data set; generalize the results to make a conclusion; compare the hypothesis and the conclusion; present the project numerically, analytically, graphically, and verbally using the predictive and analytic tools of calculus. **M.C.23**