

Utah Mathematics

# Grades 9, 10, 11, 12

Adopted 2016

1. Make sense of problems and persevere in solving them. Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method. SI.MP.1
2. Reason abstractly and quantitatively. Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem. SI.MP.2
3. Construct viable arguments and critique the reasoning of others. Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others. SI.MP.3
4. Model with mathematics. Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. SI.MP.4
5. Use appropriate tools strategically. Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts. SI.MP.5
6. Attend to precision. Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. SI.MP.6
7. Look for and make use of structure. Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. SI.MP.7

8. Look for and express regularity in repeated reasoning. Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details. **SI.MP.8**
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**Q. Number and Quantity – Quantities** **N.Q**

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **N.Q.1**
  2. Define appropriate quantities for the purpose of descriptive modeling. **N.Q.2**
  3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **N.Q.3**
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**SSE. Algebra – Seeing Structure in Expressions** **A.SSE**

1. Interpret linear expressions and exponential expressions with integer exponents that represent a quantity in terms of its context. **A.SSE.1**
    - a. Interpret parts of an expression, such as terms, factors, and coefficients. **A.SSE.1.A**
    - b. Interpret complicated expressions by viewing one or more of their parts as a single entity. **A.SSE.1.B**
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**CED. Algebra – Creating Equations** **A.CED**

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and simple exponential functions. **A.CED.1**
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A.CED.2**
3. Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. **A.CED.3**
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. **A.CED.4**

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**REI. Algebra — Reasoning With Equations and Inequalities** A.REI

1. Explain each step in solving a linear equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Students will solve exponential equations with logarithms in Secondary Mathematics III. A.REI.1
3. Solve equations and inequalities in one variable. A.REI.3
  - a. Solve one-variable equations and literal equations to highlight a variable of interest. A.REI.3.A
  - b. Solve compound inequalities in one variable, including absolute value inequalities. A.REI.3.B
  - c. Solve simple exponential equations that rely only on application of the laws of exponents (limit solving exponential equations to those that can be solved without logarithms). A.REI.3.C
5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. A.REI.5
6. Solve systems of linear equations exactly and approximately (numerically, algebraically, graphically), focusing on pairs of linear equations in two variables. A.REI.6
10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A.REI.10
11. Explain why the x-coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately; e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear and exponential functions. A.REI.11
12. Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. A.REI.12

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## IF. Functions — Interpreting Linear and Exponential Functions **F.IF**

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ . **F.IF.1**
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **F.IF.2**
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. Recognize arithmetic and geometric sequences as examples of linear and exponential functions. **F.IF.3**
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior. **F.IF.4**
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. **F.IF.5**
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **F.IF.6**
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **F.IF.7**
  - a. Graph linear functions and show intercepts. **F.IF.7.A**
  - e. Graph exponential functions, showing intercepts and end behavior. **F.IF.7.E**
9. Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). **F.IF.9**

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**BF. Functions — Building Linear or Exponential Functions** F.BF

1. Write a function that describes a relationship between two quantities. F.BF.1
  - a. Determine an explicit expression, a recursive process, or steps for calculation from a context. F.BF.1.A
  - b. Combine standard function types using arithmetic operations. F.BF.1.B
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Limit F.BF.1a, 1b, and 2 to linear and exponential functions. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions. F.BF.2
3. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ , for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Relate the vertical translation of a linear function to its  $y$ -intercept. Experiment with cases and illustrate an explanation of the effects on the graph using technology. F.BF.3

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**LE. Functions — Linear and Exponential** F.LE

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. F.LE.1
  - a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. F.LE.1.A
  - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. F.LE.1.B
  - c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.LE.1.C
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.LE.2
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly. F.LE.3
5. Interpret the parameters in a linear or exponential function in terms of a context. Limit exponential functions to those of the form  $f(x) = b \cdot \sup x + k$ . F.LE.5

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**CO. Geometry — Congruence** G.CO

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G.CO.1
2. Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). G.CO.2
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. G.CO.3
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. G.CO.4
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Point out the basis of rigid motions in geometric concepts, for example, translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle. G.CO.5
6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide whether they are congruent. G.CO.6
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G.CO.7
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. G.CO.8
12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Emphasize the ability to formalize and defend how these constructions result in the desired objects. G.CO.12
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. Emphasize the ability to formalize and defend how these constructions result in the desired objects. G.CO.13

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**GPE. Geometry — Expressing Geometric Properties With Equations** G.GPE

4. Use coordinates to prove simple geometric theorems algebraically. G.GPE.4
5. Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). G.GPE.5
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles; e.g., connect with The Pythagorean Theorem and the distance formula. G.GPE.7

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**ID. Statistics and Probability — Interpreting Categorical and Quantitative Data** S.ID

1. Represent data with plots on the real number line (dot plots, histograms, and box plots). S.ID.1
  2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S.ID.2
  3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Calculate the weighted average of a distribution and interpret it as a measure of center. S.ID.3
  6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. S.ID.6
    - a. Fit a linear function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions, or choose a function suggested by the context. Emphasize linear and exponential models. S.ID.6.A
    - b. Informally assess the fit of a function by plotting and analyzing residuals. Focus on situations for which linear models are appropriate. S.ID.6.B
    - c. Fit a linear function for scatter plots that suggest a linear association. S.ID.6.C
  7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. S.ID.7
  8. Compute (using technology) and interpret the correlation coefficient of a linear fit. S.ID.8
  9. Distinguish between correlation and causation. S.ID.9
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**VM. Number and Quantity: Vector and Matrix Quantities** *N.VM*

1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g.,  $v$ ,  $|v|$ ,  $\|v\|$ ,  $v$ ). *N.VM.1*
2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. *N.VM.2*
3. Solve problems involving velocity and other quantities that can be represented by vectors. *N.VM.3*
4. Add and subtract vectors. *N.VM.4*
  - a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. *N.VM.4.A*
  - b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. *N.VM.4.B*
  - c. Understand vector subtraction  $v - w$  as  $v + (-w)$ , where  $-w$  is the additive inverse of  $w$ , with the same magnitude as  $w$  and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. *N.VM.4.C*
5. Multiply a vector by a scalar. *N.VM.5*
  - a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as  $c(v_x, v_y) = (cv_x, cv_y)$ . *N.VM.5.A*
  - b. Compute the magnitude of a scalar multiple  $cv$  using  $\|cv\| = |c|v$ . Compute the direction of  $cv$  knowing that when  $|c|v \neq 0$ , the direction of  $cv$  is either along  $v$  (for  $c > 0$ ) or against  $v$  (for  $c < 0$ ). *N.VM.5.B*
6. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. *N.VM.6*
7. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. *N.VM.7*
8. Add, subtract, and multiply matrices of appropriate dimensions. *N.VM.8*
9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. *N.VM.9*
10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. *N.VM.10*
11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. *N.VM.11*

12. Work with  $2 \times 2$  matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. [N.VM.12](#)
  13. Solve systems of linear equations up to three variables using matrix row reduction. [N.VM.13](#)
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**MP. Mathematical Practices** MP

1. Make sense of problems and persevere in solving them. Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method. SII.MP.1
2. Reason abstractly and quantitatively. Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem. SII.MP.2
3. Construct viable arguments and critique the reasoning of others. Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others. SII.MP.3
4. Model with mathematics. Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. SII.MP.4
5. Use appropriate tools strategically. Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts. SII.MP.5
6. Attend to precision. Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. SII.MP.6
7. Look for and make use of structure. Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. SII.MP.7

8. Look for and express regularity in repeated reasoning. Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details. **S.II.MP.8**
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**RN. Number and Quantity — The Real Number System** **N.RN**

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. **N.RN.1**
  2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. **N.RN.2**
  3. Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational. Connect to physical situations (e.g., finding the perimeter of a square of area 2). **N.RN.3**
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**CN. Number and Quantity — The Complex Number System** **N.CN**

1. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real. **N.CN.1**
2. Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Limit to multiplications that involve  $i^2$  as the highest power of  $i$ . **N.CN.2**
7. Solve quadratic equations with real coefficients that have complex solutions. **N.CN.7**
8. Extend polynomial identities to the complex numbers. Limit to quadratics with real coefficients. **N.CN.8**
9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **N.CN.9**

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**SSE. Algebra — Seeing Structure in Expression** A.SSE

1. Interpret quadratic and exponential expressions that represent a quantity in terms of its context. A.SSE.1
  - a. Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.1.A
  - b. Interpret increasingly more complex expressions by viewing one or more of their parts as a single entity. Exponents are extended from the integer exponents to rational exponents focusing on those that represent square or cube roots. A.SSE.1.B
2. Use the structure of an expression to identify ways to rewrite it. A.SSE.2
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.SSE.3
  - a. Factor a quadratic expression to reveal the zeros of the function it defines. A.SSE.3.A
  - b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. A.SSE.3.B
  - c. Use the properties of exponents to transform expressions for exponential functions. A.SSE.3.C

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**APR. Algebra — Arithmetic With Polynomials and Rational Expressions** A.APR

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. A.APR.1

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**CED. Algebra — Creating Equations** A.CED

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. A.CED.1
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.2
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations; extend to formulas involving squared variables. A.CED.4

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**REI. Algebra — Reasoning With Equations and Inequalities** A.REI

4. Solve quadratic equations in one variable. A.REI.4
  - a. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form. A.REI.4.A
  - b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ . A.REI.4.B
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. A.REI.7

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**IF. Functions — Interpret Functions** F.IF

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.4
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Focus on quadratic functions; compare with linear and exponential functions. F.IF.5
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. F.IF.6
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.7
  - a. Graph linear and quadratic functions and show intercepts, maxima, and minima. F.IF.7.A
  - b. Graph piecewise-defined functions and absolute value functions. Compare and contrast absolute value and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. F.IF.7.B
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.8
  - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. F.IF.8.A
  - b. Use the properties of exponents to interpret expressions for exponential functions. F.IF.8.B
9. Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored. F.IF.9

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**BF. Functions — Building Functions** F.BF

1. Write a quadratic or exponential function that describes a relationship between two quantities. F.BF.1
  - a. Determine an explicit expression, a recursive process, or steps for calculation from a context. F.BF.1.A
  - b. Combine standard function types using arithmetic operations. F.BF.1.B
3. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Focus on quadratic functions and consider including absolute value functions. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. F.BF.3

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**LE. Functions — Linear, Quadratic, and Exponential Models** F.LE

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Compare linear and exponential growth to quadratic growth. F.LE.3

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**TF. Functions — Trigonometric Functions** F.TF

8. Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$ , given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$ , and the quadrant of the angle. F.TF.8

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**CO. Geometry — Congruence** G.CO

9. Prove theorems about lines and angles. G.CO.9
10. Prove theorems about triangles. G.CO.10
11. Prove theorems about parallelograms. G.CO.11

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**SRT. Geometry — Similarity, Right Triangles, and Trigonometry** G.SRT

1. Verify experimentally the properties of dilations given by a center and a scale factor. G.SRT.1
  - a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. G.SRT.1.A
  - b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. G.SRT.1.B
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide whether they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. G.SRT.2
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. G.SRT.3
4. Prove theorems about triangles. G.SRT.4
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. G.SRT.5
6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G.SRT.6
7. Explain and use the relationship between the sine and cosine of complementary angles. G.SRT.7
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. G.SRT.8

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**C. Geometry — Circles** G.C

1. Prove that all circles are similar. G.C.1
2. Identify and describe relationships among inscribed angles, radii, and chords. G.C.2
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. G.C.3
4. Construct a tangent line from a point outside a given circle to the circle. G.C.4
5. Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. G.C.5

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**GPE. Geometry — Expressing Geometric Properties With Equations** G.GPE

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. G.GPE.1
4. Use coordinates to prove simple geometric theorems algebraically. G.GPE.4
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. G.GPE.6

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**GMD. Geometry — Geometric Measurement and Dimension** G.GMD

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Informal arguments for area formulas can make use of the way in which area scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor  $k$ , its area is  $k^2$  times the area of the first. G.GMD.1
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. Informal arguments for volume formulas can make use of the way in which volume scale under similarity transformations: when one figure results from another by applying a similarity transformation, volumes of solid figures scale by  $k^3$  under a similarity transformation with scale factor  $k$ . G.GMD.3

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**ID. Statistics — Interpreting Categorical and Quantitative Data** S.ID

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. S.ID.5

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**CP. Statistics — Conditional Probability and the Rules of Probability** S.CP

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). S.CP.1
  4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. S.CP.4
  5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. S.CP.5
  6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. S.CP.6
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## Secondary Mathematics II — Honors Standards

### **CN. Number and Quantity — Complex Number System** N.CN

3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. N.CN.3
4. Represent complex numbers on the complex plane in rectangular form, and explain why the rectangular form of a given complex number represents the same number. N.CN.4
5. Represent addition, subtraction, and multiplication geometrically on the complex plane; use properties of this representation for computation. N.CN.5

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### **REI. Algebra — Reasoning With Equations and Inequalities** A.REI

8. Represent a system of linear equations as a single-matrix equation in a vector variable. A.REI.8
9. Find the inverse of a matrix if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension  $3 \times 3$  or greater). A.REI.9

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### **IF. Functions — Interpreting Functions** F.IF

10. Use sigma notation to represent the sum of a finite arithmetic or geometric series. F.IF.10
11. Represent series algebraically, graphically, and numerically. F.IF.11

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### **G-GPE. Geometry — Expressing Geometric Properties With Equations** G-GPE

2. Derive the equation of a parabola given a focus and directrix. G.GPE.2
3. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. G.GPE.3

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### **CP. Statistics and Probability — Conditional Probability and the Rules of Probability** S.CP

2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S.CP.2
  3. Understand the conditional probability of A given B as  $P(A \text{ and } B)/P(B)$ , and interpret independence of A and B as saying that the conditional probability of B given A is the same as the probability of B. S.CP.3
  7. Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model. S.CP.7
  8. Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$ , and interpret the answer in terms of the model. S.CP.8
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1. Make sense of problems and persevere in solving them. Explain the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships, and goals. Make conjectures about the form and meaning of the solution, plan a solution pathway, and continually monitor progress asking, "Does this make sense?" Consider analogous problems, make connections between multiple representations, identify the correspondence between different approaches, look for trends, and transform algebraic expressions to highlight meaningful mathematics. Check answers to problems using a different method. **SIII.MP.1**
2. Reason abstractly and quantitatively. Make sense of the quantities and their relationships in problem situations. Translate between context and algebraic representations by contextualizing and decontextualizing quantitative relationships. This includes the ability to decontextualize a given situation, representing it algebraically and manipulating symbols fluently as well as the ability to contextualize algebraic representations to make sense of the problem. **SIII.MP.2**
3. Construct viable arguments and critique the reasoning of others. Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Make conjectures and build a logical progression of statements to explore the truth of their conjectures. Justify conclusions and communicate them to others. Respond to the arguments of others by listening, asking clarifying questions, and critiquing the reasoning of others. **SIII.MP.3**
4. Model with mathematics. Apply mathematics to solve problems arising in everyday life, society, and the workplace. Make assumptions and approximations, identifying important quantities to construct a mathematical model. Routinely interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. **SIII.MP.4**
5. Use appropriate tools strategically. Consider the available tools and be sufficiently familiar with them to make sound decisions about when each tool might be helpful, recognizing both the insight to be gained as well as the limitations. Identify relevant external mathematical resources and use them to pose or solve problems. Use tools to explore and deepen their understanding of concepts. **SIII.MP.5**
6. Attend to precision. Communicate precisely to others. Use explicit definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Specify units of measure and label axes to clarify the correspondence with quantities in a problem. Calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. **SIII.MP.6**
7. Look for and make use of structure. Look closely at mathematical relationships to identify the underlying structure by recognizing a simple structure within a more complicated structure. See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. **SIII.MP.7**

8. Look for and express regularity in repeated reasoning. Notice if reasoning is repeated, and look for both generalizations and shortcuts. Evaluate the reasonableness of intermediate results by maintaining oversight of the process while attending to the details. [S.III.MP.8](#)
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**CN. Number and Quantity — The Complex Number System** [N.CN](#)

8. Extend polynomial identities to the complex numbers. [N.CN.8](#)
  9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Limit to polynomials with real coefficients. [N.CN.9](#)
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**SSE. Algebra — Seeing Structures in Expressions** [A.SSE](#)

1. Interpret polynomial and rational expressions that represent a quantity in terms of its context. [A.SSE.1](#)
  - a. Interpret parts of an expression, such as terms, factors, and coefficients. [A.SSE.1.A](#)
  - b. Interpret complex expressions by viewing one or more of their parts as a single entity. [A.SSE.1.B](#)
2. Use the structure of an expression to identify ways to rewrite it. [A.SSE.2](#)
4. Understand the formula for the sum of a series and use the formula to solve problems. [A.SSE.4](#)
  - a. Derive the formula for the sum of an arithmetic series. [A.SSE.4.A](#)
  - b. Derive the formula for the sum of a geometric series, and use the formula to solve problems. Extend to infinite geometric series. [A.SSE.4.B](#)

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**APR. Algebra — Arithmetic With Polynomials and Rational Expressions** A.APR

1. Understand that all polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. A.APR.1
2. Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ . A.APR.2
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. A.APR.3
4. Prove polynomial identities and use them to describe numerical relationships. A.APR.4
5. Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers. A.APR.5
6. Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division or, for the more complicated examples, a computer algebra system. A.APR.6
7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. A.APR.7

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**CED. Algebra: Creating Equations** A.CED

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. A.CED.1
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.2
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. A.CED.3
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. A.CED.4

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**REI. Algebra: Reasoning With Equations and Inequalities** A.REI

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. A.REI.2
11. Explain why the x-coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, for example, using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. A.REI.11

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**IF. Functions — Interpreting Functions** F.IF

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.4
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.IF.5
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. F.IF.6
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.7
  - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Compare and contrast square root, cubed root, and step functions with all other functions. F.IF.7.B
  - c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. F.IF.7.C
  - d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. F.IF.7.D
  - e. Graph exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude. F.IF.7.E
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.8
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). F.IF.9

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**BF. Functions — Building Functions** F.BF

1. Write a function that describes a relationship between two quantities. F.BF.1
  - b. Combine standard function types using arithmetic operations. F.BF.1.B
3. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Note the effect of multiple transformations on a single function and the common effect of each transformation across function types. Include functions defined only by a graph. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. F.BF.3
4. Find inverse functions. F.BF.4
  - a. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse. Include linear, quadratic, exponential, logarithmic, rational, square root, and cube root functions. F.BF.4.A

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**LE. Functions — Linear, Quadratic, and Exponential Models** F.LE

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. F.LE.3
4. For exponential models, express as a logarithm the solution to  $ab^{ct} = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology. Include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that  $\log xy = \log x + \log y$ . F.LE.4
5. Interpret the parameters in a linear, quadratic, and exponential functions in terms of a context. F.LE.5

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**TF. Functions — Trigonometric Functions** F.TF

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F.TF.1
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. F.TF.2
3. Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\pi - x$ ,  $\pi + x$ , and  $2\pi - x$  in terms of their values for  $x$ , where  $x$  is any real number. F.TF.3
5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. F.TF.5
7. Use inverse functions to solve trigonometric equations that arise in modeling context; evaluate the solutions using technology and interpret them in terms of context. Limit solutions to a given interval. F.TF.7

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**SRT. Geometry — Similarity, Right Triangles, and Trigonometry** G.SRT

9. Derive the formula  $A = \frac{1}{2} ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. G.SRT.9
10. Prove the Laws of Sines and Cosines and use them to solve problems. G.SRT.10
11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). G.SRT.11

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**GMD. Geometry — Geometric Measurement and Dimension** G.GMD

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of twodimensional objects. G.GMD.4

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**MG. Geometry — Modeling With Geometry** G.MG

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). G.MG.1
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). G.MG.2
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). G.MG.3

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**ID. Statistics — Interpreting Categorical and Quantitative Data** S.ID

4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. S.ID.4

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**IC. Statistics — Making Inferences and Justifying Conclusions** S.IC

1. Understand that statistics allow inferences to be made about population parameters based on a random sample from that population. S.IC.1
  3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S.IC.3
  4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. S.IC.4
  6. Evaluate reports based on data. S.IC.6
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**Secondary Mathematics  
III — Honors Standards**

**CN. Number and Quantity — Complex Number System** N.CN

3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. N.CN.3
  4. Represent complex numbers on the complex plane in rectangular form and polar form (including real and imaginary numbers), and explain why the rectangular form of a given complex number represents the same number. N.CN.4
  5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. N.CN.5
  6. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. N.CN.6
  10. Multiply complex numbers in polar form and use DeMoivre's Theorem to find roots of complex numbers. N.CN.10
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**IF. Functions — Interpreting Functions** F.IF

7. Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.7
    - d. Graph rational functions, identifying zeros, asymptotes, and point discontinuities when suitable factorizations are available, and showing end behavior. F.IF.7.D
    - f. Define a curve parametrically and draw its graph. F.IF.7.F
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**BF. Functions — Building Functions** F.BF

1. Write a function that describes a relationship between two quantities. F.BF.1
  - c. Compose functions. F.BF.1.C
4. Find inverse functions. F.BF.4
  - b. Verify by composition that one function is the inverse of another. F.BF.4.B
  - c. Read values of an inverse function from a graph or a table, given that the function has an inverse. F.BF.4.C
  - d. Produce an invertible function from a non-invertible function by restricting the domain. F.BF.4.D
5. Understand the inverse relationship between exponents and logarithms, and use this relationship to solve problems involving logarithms and exponents. F.BF.5

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**TF. Functions — Trigonometric Functions** F.TF

4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. F.TF.4
6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. F.TF.6
7. Use the inverse functions to solve trigonometric equations that arise in the modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. F.TF.7
9. Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems. F.TF.9

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**GMD. Geometry — Geometric Measurement and Dimension** G.GMD

2. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. G.GMD.2

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**CP. Statistics and Probability — Conditional Probability and the Rules of Probability** S.CP

9. Use permutations and combinations to compute probabilities of compound events and solve problems. S.CP.9

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**Precalculus****MP. Mathematical Practices** MP

1. Make sense of problems and persevere in solving them. P.MP.1
2. Reason abstractly and quantitatively. P.MP.2
3. Construct viable arguments and critique the reasoning of others. P.MP.3
4. Model with mathematics. P.MP.4
5. Use appropriate tools strategically. P.MP.5
6. Attend to precision. P.MP.6
7. Look for and make use of structure. P.MP.7
8. Look for and express regularity in repeated reasoning. P.MP.8

## VM. Number and Quantity — Vector and Matrix Quantities N.VM

1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g.,  $v$ ,  $|v|$ ,  $\|v\|$ ,  $v$ ). N.VM.1
2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. N.VM.2
3. Solve problems involving velocity and other quantities that can be represented by vectors. N.VM.3
4. Add and subtract vectors. N.VM.4
  - a. Add vectors end to end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. N.VM.4.A
  - b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. N.VM.4.B
  - c. Understand vector subtraction  $v - w$  as  $v + (-w)$ , where  $-w$  is the additive inverse of  $w$ , with the same magnitude as  $w$  and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. N.VM.4.C
5. Multiply a vector by a scalar. N.VM.5
  - a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as  $c(v_x, v_y) = (cv_x, cv_y)$ . N.VM.5.A
  - b. Compute the magnitude of a scalar multiple  $cv$  using  $\|cv\| = |c|v$ . Compute the direction of  $cv$  knowing that when  $|c|v \neq 0$ , the direction of  $cv$  is either along  $v$  (for  $c > 0$ ) or against  $v$  (for  $c < 0$ ). N.VM.5.B
6. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. N.VM.6
7. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. N.VM.7
8. Add, subtract, and multiply matrices of appropriate dimensions. N.VM.8
9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. N.VM.9
10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. N.VM.10
11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. N.VM.11

12. Work with  $2 \times 2$  matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. [N.VM.12](#)
  13. Solve systems of linear equations up to three variables using matrix row reduction. [N.VM.13](#)
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### **CN. Number and Quantity — Complex Number Systems** [N.CN](#)

3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. [N.CN.3](#)
  4. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. [N.CN.4](#)
  5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. [N.CN.5](#)
  6. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. [N.CN.6](#)
  10. Multiply complex numbers in polar form and use DeMoivre's Theorem to find roots of complex numbers. [N.CN.10](#)
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### **REI. Algebra: Reasoning With Equations and Inequalities** [A.REI](#)

8. Represent a system of linear equations as a single matrix equation in a vector variable. [A.REI.8](#)
  9. Find the inverse of a matrix, if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension  $3 \times 3$  or greater). [A.REI.9](#)
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### **IF. Functions — Interpreting Functions** [F.IF](#)

7. Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases. [F.IF.7](#)
  - d. Graph rational functions, identifying zeros, asymptotes, and point discontinuities when suitable factorizations are available, and showing end behavior. [F.IF.7.D](#)
  - f. Define a curve parametrically and draw its graph. [F.IF.7.F](#)
10. Use sigma notation to represent the sum of a finite arithmetic or geometric series. [F.IF.10](#)
11. Represent series algebraically, graphically, and numerically. [F.IF.11](#)

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**BF. Functions — Building Functions** F.BF

1. Write a function that describes a relationship between two quantities. F.BF.1
  - c. Compose functions. F.BF.1.C
4. Find inverse functions. F.BF.4
  - b. Verify by composition that one function is the inverse of another. F.BF.4.B
  - c. Read values of an inverse function from a graph or a table, given that the function has an inverse. F.BF.4.C
  - d. Produce an invertible function from a non-invertible function by restricting the domain. F.BF.4.D
5. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. F.BF.5

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**TF. Functions — Trigonometric Functions** F.TF

4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. F.TF.4
6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. F.TF.6
7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context F.TF.7
9. Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems. F.TF.9

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**GMD. Geometry — Geometric Measurement and Dimension** G.GMD

2. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. G.GMD.2

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**GPE. Geometry — Expressing Geometric Properties With Equations** G.GPE

2. Derive the equation of a parabola given a focus and a directrix. G.GPE.2
3. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. G.GPE.3

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**CP. Statistics — Conditional Probability and the Rules of Probability** S.CP

2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S.CP.2
3. Understand the conditional probability of A given B as  $P(A \text{ and } B)/P(B)$ , and interpret independence of A and B as saying that the conditional probability of B given A is the same as the probability of B. S.CP.3
7. Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model. S.CP.7
8. Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$ , and interpret the answer in terms of the model. S.CP.8
9. Use permutations and combinations to compute probabilities of compound events and solve problems. S.CP.9