Tennessee Mathematics

Integrated Math III

Adopted 2023

Algebra I

Number and Quantity

Quantities

A. Reason quantitatively and use units to understand problems. A1.N.Q.A

1. Use units as a way to understand real-world problems. A1.N.Q.A.1

a. Choose and interpret the scale and the origin in graphs and data displays, A1.N.Q.A.1.A

b. Use appropriate quantities in formulas, converting units as necessary. A1.N.Q.A.1.B

c. Define and justify appropriate quantities within a context for the purpose of modeling. A1.N.Q.A.1.C

d. Choose an appropriate level of accuracy when reporting quantities. A1.N.Q.A.1.D

Algebra

Seeing Structure in Expressions

A. Interpret the structure of expressions. A1.A.SSE.A

1. Interpret expressions that represent a quantity in terms of its context. A1.A.SSE.A.1

    a. Interpret parts of an expression, such as terms, factors, and coefficients. A1.A.SSE.A.1.A

    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. A1.A.SSE.A.1.B

Arithmetic with Polynomials and Rational Expressions

A. Perform arithmetic operations on polynomials. A1.A.APR.A

1. Add, subtract, and multiply polynomials. Use these operations to demonstrate that polynomials form a closed system that adhere to the same properties of operations as the integers. A1.A.APR.A.1

Creating Equations

A. Create equations that describe numbers or relationships. A1.A.CED.A

1. Create equations and inequalities in one variable and use them to solve problems in a real-world context. A1.A.CED.A.1

2. Create equations and inequalities in two variables to represent relationships between quantities and use them to solve problems in a real-world context. Graph equations with two variables on coordinate axes with labels and scales, and use the graphs to make predictions. A1.A.CED.A.2

3. Create individual and systems of equations and/or inequalities to represent constraints in a contextual situation, and interpret solutions as viable or non-viable. A1.A.CED.A.3

4. Rearrange formulas to isolate a quantity of interest using algebraic reasoning. A1.A.CED.A.4

Reasoning with Equations and Inequalities

A. Understand solving equations as a process of reasoning and explain the reasoning. A1.A.REI.A

1. Understand solving equations as a process of reasoning and explain the reasoning. Construct a viable argument to justify a solution method. A1.A.REI.A.1

B. Solve equations and inequalities in one variable. A1.A.REI.B

2. Solve linear and absolute value equations and inequalities in one variable. A1.A.REI.B.2

    a. Solve linear equations and inequalities, including compound inequalities, in one variable. Represent solutions algebraically and graphically. A1.A.REI.B.2.A

b. Solve absolute value equations and inequalities in one variable. Represent solutions algebraically and graphically. A1.A.REI.B.2.B

3. Solve quadratic equations and inequalities in one variable. A1.A.REI.B.3

a. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when a quadratic equation has solutions that are not real numbers. A1.A.REI.B.3.A

b. Solve quadratic inequalities using the graph of the related quadratic equation. A1.A.REI.B.3.B

C. Solve systems of equations. A1.A.REI.C

4. Write and solve a system of linear equations in real-world context. A1.A.REI.C.4

D. Represent and solve equations and inequalities graphically. A1.A.REI.D

5. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A1.A.REI.D.5

6. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. Find approximate solutions by graphing the functions or making a table of values, using technology when appropriate. A1.A.REI.D.6

7. Graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. A1.A.REI.D.7

Functions

Interpreting Functions

A. Understand the concept of a function and use function notation. A1.F.IF.A

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). A1.F.IF.A.1
2. Use function notation. A1.F.IF.A.2
   a. Use function notation to evaluate functions for inputs in their domains, including functions of two variables. A1.F.IF.A.2.A
   b. Interpret statements that use function notation in terms of a context. A1.F.IF.A.2.B
3. Understand geometric formulas as functions. A1.F.IF.A.3

B. Interpret functions that arise in applications in terms of the context. A1.F.IF.B

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. A1.F.IF.B.4
5. Relate the domain of a function to its graph and, where applicable, to the context of the function it models. A1.F.IF.B.5
6. Calculate and interpret the average rate of change of a function (presented algebraically or as a table) over a specified interval. Estimate and interpret the rate of change from a graph. A1.F.IF.B.6

C. Analyze functions using different representations. A1.F.IF.C

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. A1.F.IF.C.8
   a. Rewrite quadratic functions to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a real-world context. A1.F.IF.C.8.A
9. Compare properties of functions represented algebraically, graphically, numerically in tables, or by verbal descriptions. A1.F.IF.C.9
   a. Compare properties of two different functions. Functions may be of different types and/or represented in different ways. A1.F.IF.C.9.A
   b. Compare properties of the same function on two different intervals or represented in two different ways. A1.F.IF.C.9.B

Building Functions

A. Build a function that models a relationship between two quantities. A1.F.BF.A

1. Build a function that describes a relationship between two quantities. A1.F.BF.A.1

a. Determine steps for calculation, a recursive process, or an explicit expression from a context. A1.F.BF.A.1.A

B. Build new functions from existing functions. A1.F.BF.B

2. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given graphs. A1.F.BF.B.2

Linear and Exponential Functions

A. Construct and compare linear and exponential models and solve problems. A1.F.LE.A

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. A1.F.LE.A.1

a. Know that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. A1.F.LE.A.1.A

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. A1.F.LE.A.1.B

c. Recognize situations in which a quantity grows or decays by a constant factor per unit interval relative to another. A1.F.LE.A.1.C

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs. A1.F.LE.A.2

B. Interpret expressions for functions in terms of the situation they model. A1.F.LE.B

3. Interpret the parameters in a linear or exponential function in terms of a context. A1.F.LE.B.3

Statistics and Probability

Interpreting Categorical and Quantitative Data

A. Summarize, represent, and interpret data on a single count or measurement variable. A1.S.ID.A

1. Use measures of center to solve real world and mathematical problems. A1.S.ID.A.1
2. Use statistics appropriate to the shape of the data distribution to compare center (mean, median, and/or mode) and spread (range, interquartile range) of two or more different data sets. A1.S.ID.A.2
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points. A1.S.ID.A.3

B. Summarize, represent, and interpret data on two categorical and quantitative variables. A1.S.ID.B

4. Represent data from two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. A1.S.ID.B.4

C. Interpret linear models. A1.S.ID.C

5. Interpret the rate of change and the constant term of a linear model in the context of data. A1.S.ID.C.5
6. Use technology to compute the correlation coefficient of a linear model; interpret the correlation coefficient in the context of the data. A1.S.ID.C.6
7. Explain the differences between correlation and causation. Recognize situations where an additional factor may be affecting correlated data. A1.S.ID.C.7

Geometry

Number and Quantity

Quantities

A. Reason quantitatively and use units to solve problems. G.N.Q.A

1. Use units as a way to understand real world problems. G.N.Q.A.1
   a. Use appropriate quantities in formulas, converting units as necessary. G.N.Q.A.1.A
   b. Define and justify appropriate quantities within a context for the purpose of modeling. G.N.Q.A.1.B
   c. Choose an appropriate level of accuracy when reporting quantities. G.N.Q.A.1.C

Geometry

Congruence

A. Experiment with transformations in the plane. G.CO.A

1. Describe transformations as functions that take points in the plane (pre-image) as inputs and give other points (image) as outputs. Compare transformations that preserve distance and angle measure to those that do not, by hand for basic transformations and using technology for more complex cases. G.CO.A.1
2. Given a rectangle, parallelogram, trapezoid, or regular polygon, determine the transformations that carry the shape onto itself and describe them in terms of the symmetry of the figure. G.CO.A.2
3. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. G.CO.A.3
4. Given a geometric figure, draw the image of the figure after a sequence of one or more rigid motions, by hand and using technology. Identify a sequence of rigid motions that will carry a given figure onto another. G.CO.A.4

B. Understand congruence in terms of rigid motions. G.CO.B

5. Given two figures, use the definition of congruence in terms of rigid motions to determine informally if they are congruent. G.CO.B.5
6. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G.CO.B.6
7. Explain how the criteria for triangle congruence (ASA, SAS, AAS, SSS, and HL) follow from the definition of congruence in terms of rigid motions. G.CO.B.7

C. Use geometric theorems to justify relationships. G.CO.C

8. Use definitions and theorems about lines and angles to solve problems and to justify relationships in geometric figures. G.CO.C.8
9. Use definitions and theorems about triangles to solve problems and to justify relationships in geometric figures. G.CO.C.9
10. Use definitions and theorems about parallelograms to solve problems and to justify relationships in geometric figures. G.CO.C.10

D. Perform geometric constructions. G.CO.D

11. Perform formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). G.CO.D.11
12. Use geometric constructions to solve geometric problems in context, by hand and using technology. G.CO.D.12

Similarity, Right Triangles, and Trigonometry

A. Understand similarity in terms of similarity transformations. G.SRT.A

1. Use properties of dilations given by a center and a scale factor to solve problems and to justify relationships in geometric figures. G.SRT.A.1
2. Define similarity in terms of transformations. Use transformations to determine whether two figures are similar. G.SRT.A.2

B. Use similarity to solve problems and justify relationships. G.SRT.B

3. Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures. G.SRT.B.3

C. Define trigonometric ratios and solve problems involving triangles. G.SRT.C

4. Use side ratios in right triangles to define trigonometric ratios. G.SRT.C.4
    a. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G.SRT.C.4.A
    b. Explain and use the relationship between the sine and cosine of complementary angles. G.SRT.C.4.B
5. Solve triangles. G.SRT.C.5
    a. Know and use the Pythagorean Theorem and trigonometric ratios (sine, cosine, tangent, and their inverses) to solve right triangles in a real-world context. G.SRT.C.5.A
    b. Know and use relationships within special right triangles to solve problems in a real-world context. G.SRT.C.5.B
    c. Use the Law of Sines and Law of Cosines to solve non-right triangles in a real-world context. G.SRT.C.5.C

Circles

A. Find areas of sectors of circles. G.C.A

1. Use proportional relationships between the area of a circle and the area of a sector within the circle to solve problems in a real-world context. G.C.A.1

Expressing Geometric Properties with Equations

A. Use coordinates to solve problems and justify simple geometric theorems algebraically. G.GPE.A

1. Use coordinates to justify geometric relationships algebraically and to solve problems. G.GPE.A.1
2. Use the slope criteria for parallel and perpendicular lines to solve problems and to justify relationships in geometric figures. G.GPE.A.2
3. Understand the relationship between the Pythagorean Theorem and the distance formula and use an efficient method to solve problems on the coordinate plane. G.GPE.A.3

Geometric Measurement and Dimension

A. Explain volume and surface area formulas and use them to solve problems. G.GMD.A

1. Understand and explain the formulas for the volume and surface area of a cylinder, cone, prism, and pyramid. G.GMD.A.1
2. Use volume and surface area formulas for cylinders, cones, prisms, pyramids, and spheres to solve problems in a real-world context. G.GMD.A.2

Modeling with Geometry

A. Apply geometric concepts in modeling situations. G.MG.A
   1. Use geometric shapes, their measures, and their properties to model objects found in a real-world context for the purpose of approximating solutions to problems. G.MG.A.1

Statistics and Probability

Conditional Probability and the Rules of Probability

A. Understand independence and conditional probability and use them to create visual representations of data. G.S.CP.A
   1. Use set notation to represent contextual situations. G.S.CP.A.1
      a. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or", "and", "not"). G.S.CP.A.1.A
      b. Flexibly move between visual models (Venn diagrams, frequency tables, etc.) and set notation. G.S.CP.A.1.B

B. Use the rules of probability to compute probabilities of compound events in a uniform probability model. G.S.CP.B
   2. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the given context. G.S.CP.B.2
   3. Understand and apply the Addition Rule. G.S.CP.B.3
      a. Explain the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B) in terms of visual models (Venn diagrams, frequency tables, etc.). G.S.CP.B.3.A
      b. Apply the Addition Rule to solve problems and interpret the answer in terms of the given context. G.S.CP.B.3.B

C. Apply geometric concepts to situations involving probability. G.S.CP.C
   4. Calculate probabilities using geometric figures. G.S.CP.C.4

Algebra II

Number and Quantity

The Real Number System

A. Extend the properties of exponents to rational exponents. A2.N.RN.A

1. Extend the properties of integer exponents to rational exponents. A2.N.RN.A.1
    a. Develop the meaning of rational exponents by applying the properties of integer exponents. A2.N.RN.A.1.A
    b. Explain why x^{1/n} can be written as the nth root of x. A2.N.RN.A.1.B
    c. Rewrite expressions involving radicals and rational exponents using the properties of exponents. A2.N.RN.A.1.C

Quantities

A. Reason quantitatively and use units to understand problems. A2.N.Q.A

1. Use units as a way to understand real-world problems. A2.N.Q.A.1
    a. Choose and interpret the scale and the origin in graphs and data displays. A2.N.Q.A.1.A
    b. Use appropriate quantities in formulas, converting units as necessary. A2.N.Q.A.1.B
    c. Define and justify appropriate quantities within a context for the purpose of modeling. A2.N.Q.A.1.C
    d. Choose an appropriate level of accuracy when reporting quantities. A2.N.Q.A.1.D

Matrices

A. Perform operations on matrices and use matrices in applications. A2.N.M.A

1. Use matrices to represent data in a real-world context. Interpret rows, columns, and dimensions of matrices in terms of the context. A2.N.M.A.1
2. Perform operations on matrices in a real-world context. A2.N.M.A.2
    a. Multiply a matrix by a scalar to produce a new matrix. A2.N.M.A.2.A
    b. Add and/or subtract matrices by hand and using technology. A2.N.M.A.2.B
    c. Multiply matrices of appropriate dimensions, by hand in simple cases and using technology for more complicated cases. A2.N.M.A.2.C
    d. Describe the roles that zero matrices and identity matrices play in matrix addition and multiplication, recognizing that they are similar to the roles of 0 and 1 in the real number system. A2.N.M.A.2.D
3. Create and use augmented matrices to solve systems of linear equations in real-world contexts, by hand and using technology. A2.N.M.A.3

Algebra

Seeing Structure in Expressions

A. Interpret the structure of expressions. A2.A.SSE.A

1. Interpret expressions that represent a quantity in terms of its context. A2.A.SSE.A.1
    a. Interpret parts of an expression, such as terms, factors, and coefficients. A2.A.SSE.A.1.A
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. A2.A.SSE.A.1.B

A. Understand the relationship between zeros and factors of polynomials. A2.A.APR.A

1. Know and apply the Factor Theorem: For a polynomial p(x) and a number a, p(a) = 0 if and only if (x – a) is a factor of p(x). A2.A.APR.A.1
2. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. A2.A.APR.A.2

Creating Equations

A. Create equations that describe numbers or relationships. A2.A.CED.A

1. Create equations and inequalities in one variable and use them to solve problems in a real-world context. A2.A.CED.A.1
2. Create equations and inequalities in two variables to represent relationships between quantities and use them to solve problems in a real-world context. Graph equations and inequalities with two variables on coordinate axes with labels and scales, and use the graphs to make predictions. A2.A.CED.A.2
3. Rearrange formulas to isolate a quantity of interest using algebraic reasoning. A2.A.CED.A.3

Reasoning with Equations and Inequalities

A. Understand solving equations as a process of reasoning and explain the reasoning. A2.A.REI.A

1. Understand solving equations as a process of reasoning and explain the reasoning. Construct a viable argument to justify a solution method. A2.A.REI.A.1
2. Solve radical equations in one variable, and identify extraneous solutions when they exist. A2.A.REI.A.2

B. Solve systems of equations. A2.A.REI.B

3. Write and solve a system of linear equations in a real-world context. A2.A.REI.B.3
4. Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically, graphically, and using technology. A2.A.REI.B.4

Functions

Interpreting Functions

A. Interpret functions that arise in applications in terms of the context. A2.F.IF.A

1. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. A2.F.IF.A.1
2. Calculate and interpret the average rate of change of a function (presented algebraically or as a table) over a specified interval. Estimate and interpret the rate of change from a graph. A2.F.IF.A.2
3. Understand geometric formulas as functions. A2.F.IF.A.3

B. Analyze functions using different representations. A2.F.IF.B

4. Graph functions expressed algebraically and show key features of the graph by hand and using technology. A2.F.IF.B.4
5. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. A2.F.IF.B.5
   a. Rewrite quadratic functions to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a real-world context. A2.F.IF.B.5.A
   b. Know and use the properties of exponents to interpret expressions for exponential functions in terms of a real-world context. A2.F.IF.B.5.B
6. Compare properties of functions represented algebraically, graphically, numerically in tables, or by verbal descriptions. A2.F.IF.B.6
   a. Compare properties of two different functions. Functions may be of different types and/or represented in different ways. A2.F.IF.B.6.A
   b. Compare properties of the same function on two different intervals or represented in two different ways. A2.F.IF.B.6.B

Building Functions

A. Build a function that models a relationship between two quantities. A2.F.BF.A

1. Build a function that describes a relationship between two quantities. A2.F.BF.A.1
   a. Combine standard function types using arithmetic operations. A2.F.BF.A.1.A
   b. Combine standard function types using composition. A2.F.BF.A.1.B
2. Define sequences as functions, including recursive definitions, whose domain is a subset of the integers. Write explicit and recursive formulas for arithmetic and geometric sequences in context and connect them to linear and exponential functions. A2.F.BF.A.2

B. Build new functions from existing functions. A2.F.BF.B

3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. A2.F.BF.B.3
    a. Determine whether a function is one-to-one. A2.F.BF.B.3.A
    b. Find the inverse of a function on an appropriate domain. A2.F.BF.B.3.B
    c. Given an invertible function on an appropriate domain, identify the domain of the inverse function. A2.F.BF.B.3.C
4. Find the inverse of a function. A2.F.BF.B.4

Linear, Quadratic, and Exponential Models

A. Construct and compare linear, quadratic, and exponential models and solve problems. A2.F.LE.A
    1. Know the relationship between exponential functions and logarithmic functions. A2.F.LE.A.1
        a. Solve exponential equations using a variety of strategies, including logarithms. A2.F.LE.A.1.A
        b. Understand that a logarithm is the solution to ab^{ct} = d, where a, b, c, and d are numbers. A2.F.LE.A.1.B
        c. Evaluate logarithms using technology. A2.F.LE.A.1.C
    2. Know that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or cubically. A2.F.LE.A.2

Statistics and Probability

Interpreting Categorical and Quantitative Data

A. Summarize, represent, and interpret data on a single count or measurement variable. A2.S.ID.A

1. Use statistics appropriate to the shape of the data distribution to compare center (mean, median, and/or mode) and spread (range, standard deviation) of two or more different data sets. A2.S.ID.A.1
2. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages using the Empirical Rule. A2.S.ID.A.2
3. Compute, interpret, and compare z-scores for normally distributed data in a real-world context. A2.S.ID.A.3

B. Summarize, represent, and interpret data on two categorical and quantitative variables. A2.S.ID.B

4. Represent data from two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. A2.S.ID.B.4

Making Inferences and Justifying Conclusions

A. Make inferences and justify conclusions from sample surveys, experiments, and observational studies. A2.S.IC.A

1. Recognize the purposes of and differences among sample surveys, experiments, and observational studies. A2.S.IC.A.1
2. Identify potential sources of bias in statistical studies. A2.S.IC.A.2
3. Distinguish between a statistic and a parameter. Evaluate reports based on data and recognize when poor conclusions are drawn from well-collected data. A2.S.IC.A.3

Conditional Probability and the Rules of Probability

A. Understand independence and conditional probability and use them to create visual representations of data. A2.S.CP.A

1. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. Categorize events as independent or dependent. A2.S.CP.A.1

B. Understand and apply basic concepts of probability. A2.S.CP.B

2. Apply statistical counting techniques. A2.S.CP.B.2
    a. Use the Fundamental Counting Principle to compute probabilities of compound events and solve problems. A2.S.CP.B.2.A
    b. Use permutations and combinations to compute probabilities of compound events and solve problems. A2.S.CP.B.2.B
3. Use the Law of Large Numbers to assess the validity of a statistical claim. A2.S.CP.B.3

C. Use the rules of probability to compute probabilities of compound events in a uniform probability model. A2.S.CP.C

4. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the given context. A2.S.CP.C.4

Integrated Math I

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Quantities

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a. Choose and interpret the scale and the origin in graphs and data displays. M1.N.Q.A.1.A

b. Use appropriate quantities in formulas, converting units as necessary. M1.N.Q.A.1.B

c. Define and justify appropriate quantities within a context for the purpose of modeling. M1.N.Q.A.1.C

d. Choose an appropriate level of accuracy when reporting quantities. M1.N.Q.A.1.D

Matrices

A. Perform operations on matrices and use matrices in applications. M1.N.M.A

1. Use matrices to represent data in a real-world context. Interpret rows, columns, and dimensions of matrices in terms of the context. M1.N.M.A.1

2. Perform operations on matrices in a real-world context. M1.N.M.A.2

a. Multiply a matrix by a scalar to produce a new matrix. M1.N.M.A.2.A

b. Add and/or subtract matrices by hand and using technology. M1.N.M.A.2.B

c. Multiply matrices of appropriate dimensions, by hand in simple cases and using technology for more complicated cases. M1.N.M.A.2.C

d. Describe the roles that zero matrices and identity matrices play in matrix addition and multiplication, recognizing that they are similar to the roles of 0 and 1 in the real number system. M1.N.M.A.2.D

3. Create and use augmented matrices to solve systems of linear equations in real-world contexts, by hand and using technology. M1.N.M.A.3

Algebra

Seeing Structure in Expressions

A. Interpret the structure of expressions. M1.A.SSE.A

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   a. quantity in terms of its context. M1.A.SSE.A.1.A

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Reasoning with Equations and Inequalities

A. Understand solving equations as a process of reasoning and explain the reasoning. M1.A.REI.A

1. Understand solving equations as a process of reasoning and explain the reasoning. Construct a viable argument to justify a solution method. M1.A.REI.A.1

B. Solve equations and inequalities in one variable. M1.A.REI.B

2. Solve linear and absolute value equations and inequalities in one variable. M1.A.REI.B.2

   a. Solve linear equations and inequalities, including compound inequalities, in one variable. Represent solutions algebraically and graphically. M1.A.REI.B.2.A

   b. Solve absolute value equations and inequalities in one variable. Represent solutions algebraically and graphically. M1.A.REI.B.2.B

C. Solve systems of equations. M1.A.REI.C

3. Write and solve a system of linear equations in a real-world context. M1.A.REI.C.3

D. Represent and solve equations and inequalities graphically. M1.A.REI.D

4. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). M1.A.REI.D.4

5. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x). Find approximate solutions by graphing the functions or making a table of values, using technology when appropriate. M1.A.REI.D.5

6. Graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding halfplanes. M1.A.REI.D.6

Functions

Interpreting Functions

A. Understand the concept of a function and use function notation. M1.F.IF.A

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). M1.F.IF.A.1
2. Use function notation. M1.F.IF.A.2
    a. Use function notation to evaluate functions for inputs in their domains, including functions of two variables. M1.F.IF.A.2.A
    b. Interpret statements that use function notation in terms of a context. M1.F.IF.A.2.B
3. Understand geometric formulas as functions. M1.F.IF.A.3

B. Interpret functions that arise in applications in terms of the context. M1.F.IF.B

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. M1.F.IF.B.4
5. Relate the domain of a function to its graph and, where applicable, to the context of the function it models. M1.F.IF.B.5

C. Analyze functions using different representations. M1.F.IF.C

6. Compare properties of functions represented algebraically, graphically, numerically in tables, or by verbal descriptions. M1.F.IF.C.6
    a. Compare properties of two different functions. Functions may be of different types and/or represented in different ways. M1.F.IF.C.6.A
    b. Compare properties of the same function on two different intervals or represented in two different ways. M1.F.IF.C.6.B

Building Functions

A. Build a function that models a relationship between two quantities. M1.F.BF.A

1. Build a function that describes a relationship between two quantities. M1.F.BF.A.1
    a. Determine steps for calculation, a recursive process, or an explicit expression from a context. M1.F.BF.A.1.A
2. Define sequences as functions, including recursive definitions, whose domain is a subset of the integers. Write explicit and recursive formulas for arithmetic and geometric sequences in context and connect them to linear and exponential functions. M1.F.BF.A.2

Linear and Exponential Models

A. Construct and compare linear and exponential models and solve problems. M1.F.LE.A

   1. Distinguish between situations that can be modeled with linear functions and with exponential functions. M1.F.LE.A.1

      a. Know that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. M1.F.LE.A.1.A

      b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. M1.F.LE.A.1.B

      c. Recognize situations in which a quantity grows or decays by a constant factor per unit interval relative to another. M1.F.LE.A.1.C

   2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a table, a description of a relationship, or input-output pairs. M1.F.LE.A.2

B. Interpret expressions for functions in terms of the situation they model. M1.F.LE.B

   3. Interpret the parameters in a linear or exponential function in terms of a context. M1.F.LE.B.3

Geometry

Congruence

A. Experiment with transformations in the plane. M1.G.CO.A

1. Describe transformations as functions that take points in the plane (pre-image) as inputs and give other points (image) as outputs. Compare transformations that preserve distance and angle measure to those that do not, by hand for basic transformations and using technology for more complex cases. M1.G.CO.A.1
2. Given a rectangle, parallelogram, trapezoid, or regular polygon, determine the transformations that carry the shape onto itself and describe them in terms of the symmetry of the figure. M1.G.CO.A.2

B. Use geometric theorems to justify relationships. M1.G.CO.B

3. Use definitions and theorems about lines and angles to solve problems and to justify relationships in geometric figures. M1.G.CO.B.3
4. Use definitions and theorems about triangles to solve problems and to justify relationships in geometric figures. M1.G.CO.B.4

C. Perform geometric constructions. M1.G.CO.C

5. Perform formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). M1.G.CO.C.5
6. Use geometric constructions to solve geometric problems in context, by hand and using technology. M1.G.CO.C.6

Geometric Properties with Equations

A. Use coordinates to solve problems and justify simple geometric theorems algebraically. M1.G.GPE.A

1. Use coordinates to solve problems and justify geometric relationships algebraically. M1.G.GPE.A.1
2. Use the slope criteria for parallel and perpendicular lines to solve problems and to justify relationships in geometric figures. M1.G.GPE.A.2
3. Understand the relationship between the Pythagorean Theorem and the distance formula and use an efficient method to solve problems on the coordinate plane. M1.G.GPE.A.3

Statistics and Probability

Interpreting Categorical and Quantitative Data

A. Summarize, represent, and interpret data on two categorical and quantitative variables. M1.S.ID.A

1. Represent data from two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. M1.S.ID.A.1

B. Interpret linear models. M1.S.ID.B

2. Interpret the rate of change and the constant term of a linear model in the context of the data. M1.S.ID.B.2

3. Use technology to compute the correlation coefficient of a linear model; interpret the correlation coefficient in the context of the data. M1.S.ID.B.3

4. Explain the differences between correlation and causation. Recognize situations where an additional factor may be affecting correlated data. M1.S.ID.B.4

Integrated Math II

Number and Quantity

The Real Number System

A. Extend the properties of exponents to rational exponents. M2.N.RN.A

1. Extend the properties of integer exponents to rational exponents. M2.N.RN.A.1

a. Develop the meaning of rational exponents by applying the properties of integer exponents. M2.N.RN.A.1.A

b. Explain why x^{1/n} can be written as the nth root of x. M2.N.RN.A.1.B

c. Rewrite expressions involving radicals and rational exponents using the properties of exponents. M2.N.RN.A.1.C

Quantities

A. Reason quantitatively and use units to understand problems. M2.N.Q.A

1. Use units as a way to understand real-world problems. M2.N.Q.A.1

a. Choose and interpret the scale and the origin in graphs and data displays. M2.N.Q.A.1.A

b. Use appropriate quantities in formulas, converting units as necessary. M2.N.Q.A.1.B

c. Define and justify appropriate quantities within a context for the purpose of modeling. M2.N.Q.A.1.C

d. Choose an appropriate level of accuracy when reporting quantities. M2.N.Q.A.1.D

Algebra

Seeing Structure in Expressions

A. Interpret the structure of expressions. M2.A.SSE.A

1. Interpret expressions that represent a quantity in terms of its context. M2.A.SSE.A.1
    a. Interpret parts of an expression, such as terms, factors, and coefficients. M2.A.SSE.A.1.A
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. M2.A.SSE.A.1.B

Arithmetic with Polynomials and Rational Expressions

A. Perform arithmetic operations on polynomials. M2.A.APR.A

1. Add, subtract, and multiply polynomials. Use these operations to demonstrate that polynomials form a closed system that adhere to the same properties of operations as the integers. M2.A.APR.A.1

B. Understand the relationship between zeros and factors of polynomials. M2.A.APR.B

2. Know and apply the Factor Theorem: For a polynomial p(x) and a number a, p(a) = 0 if and only if (x – a) is a factor of p(x). M2.A.APR.B.2

Creating Equations

A. Create equations that describe numbers or relationships. M2.A.CED.A

1. Create equations and inequalities in one variable and use them to solve problems in a real-world context. M2.A.CED.A.1
2. Create equations and inequalities in two variables to represent relationships between quantities and use them to solve problems in a real world context. Graph equations with two variables on coordinate axes with labels and scales, and use the graphs to make predictions. M2.A.CED.A.2
3. Rearrange formulas to isolate a quantity of interest using algebraic reasoning. M2.A.CED.A.3

Reasoning with Equations and Inequalities

A. Understand solving equations as a process of reasoning and explain the reasoning. M2.A.REI.A

1. Understand solving equations as a process of reasoning and explain the reasoning. Construct a viable argument to justify a solution method. M2.A.REI.A.1

B. Solve equations and inequalities in one variable. M2.A.REI.B

2. Solve quadratic equations and inequalities in one variable. M2.A.REI.B.2
    a. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, knowing and applying the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when a quadratic equation has nonreal solutions. M2.A.REI.B.2.A

b. Solve quadratic inequalities using the graph of the related quadratic equation. M2.A.REI.B.2.B

3. Solve radical equations in one variable and identify extraneous solutions when they exist. M2.A.REI.B.3

C. Solve systems of equations. M2.A.REI.C

4. Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically, graphically, and using technology. M2.A.REI.C.4

D. Represent and solve equations and inequalities graphically. M2.A.REI.D

5. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. Find approximate solutions by graphing the functions or making a table of values, using technology when appropriate. M2.A.REI.D.5

Functions

Interpreting Functions

A. Understand the concept of function and use function notation. M2.F.IF.A

1. Use function notation. M2.F.IF.A.1

   a. Use function notation to evaluate functions for inputs in their domains, including functions of two variables. M2.F.IF.A.1.A

   b. Interpret statements that use function notation in terms of a context. M2.F.IF.A.1.B

2. Understand geometric formulas as functions. M2.F.IF.A.2

B. Interpret functions that arise in applications in terms of the context. M2.F.IF.B

3. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. M2.F.IF.B.3

4. Relate the domain of a function to its graph and, where applicable, to the context of the function it models. M2.F.IF.B.4

5. Calculate and interpret the average rate of change of a function (presented algebraically or as a table) over a specified interval. Estimate and interpret the rate of change from a graph. M2.F.IF.B.5

C. Analyze functions using different representation. M2.F.IF.C

6. Graph functions expressed algebraically and show key features of the graph by hand and using technology. M2.F.IF.C.6

7. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. M2.F.IF.C.7

   a. Rewrite quadratic functions to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a real-world context. M2.F.IF.C.7.A

   b. Know and use the properties of exponents to interpret expressions for exponential functions in terms of a real-world context. M2.F.IF.C.7.B

8. Compare properties of functions represented algebraically, graphically, numerically in tables, or by verbal descriptions. M2.F.IF.C.8

   a. Compare properties of two different functions. Functions may be of different types and/or represented in different ways. M2.F.IF.C.8.A

   b. Compare properties of the same function on two different intervals or represented in two different ways. M2.F.IF.C.8.B

Building Functions

A. Build a function that models a relationship between two quantities. M2.F.BF.A

1. Build a function that describes a relationship between two quantities. M2.F.BF.A.1

a. Combine standard function types using arithmetic operations. M2.F.BF.A.1.A

B. Build new functions from existing functions. M2.F.BF.B

2. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given graphs. M2.F.BF.B.2

Geometry

Congruence

A. Experiment with transformations in the plane. M2.G.CO.A

1. Describe transformations as functions that take points in the plane (pre-image) as inputs and give other points (image) as outputs. Compare transformations that preserve distance and angle measure to those that do not, by hand for basic transformations and using technology for more complex cases. M2.G.CO.A.1
2. Given a rectangle, parallelogram, trapezoid, or regular polygon, determine the transformations that carry the shape onto itself and describe them in terms of the symmetry of the figure. There are no assessment limits for this standard. The entire standard is assessed in this course. M2.G.CO.A.2
3. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. There are no assessment limits for this standard. The entire standard is assessed in this course. M2.G.CO.A.3
4. Given a geometric figure, draw the image of the figure after a sequence of one or more rigid motions, by hand and using technology. Identify a sequence of rigid motions that will carry a given figure onto another. M2.G.CO.A.4

B. Understand congruence in terms of rigid motions. M2.G.CO.B

2. Given a rectangle, parallelogram, trapezoid, or regular polygon, determine the transformations that carry the shape onto itself and describe them in terms of the symmetry of the figure. M2.G.CO.B.2
3. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. M2.G.CO.B.3
4. Given a geometric figure, draw the image of the figure after a sequence of one or more rigid motions, by hand and using technology. Identify a sequence of rigid motions that will carry a given figure onto another. M2.G.CO.B.4
5. Given two figures, use the definition of congruence in terms of rigid motions to determine informally if they are congruent. M2.G.CO.B.5
6. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. M2.G.CO.B.6
7. Explain how the criteria for triangle congruence (ASA, SAS, AAS, SSS, and HL) follow from the definition of congruence in terms of rigid motions. M2.G.CO.B.7

C. Use geometric theorems to justify relationships. M2.G.CO.C

8. Use definitions and theorems about triangles to solve problems and to justify relationships in geometric figures. M2.G.CO.C.8
9. Use definitions and theorems about parallelograms to solve problems and to justify relationships in geometric figures. M2.G.CO.C.9

Similarity, Right Triangles, and Trigonometry

A. Understand similarity in terms of similarity transformations. M2.G.SRT.A

1. Use properties of dilations given by a center and a scale factor to solve problems and to justify relationships in geometric figures. M2.G.SRT.A.1
2. Define similarity in terms of transformations. Use transformations to determine whether two figures are similar. M2.G.SRT.A.2

B. Use similarity to solve problems and justify relationships. M2.G.SRT.B

3. Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures. M2.G.SRT.B.3

Statistics and Probability

Interpreting Categorical and Quantitative Data

A. Summarize, represent, and interpret data on two categorical and quantitative variables. M2.S.ID.A

1. Represent data from two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. M2.S.ID.A.1

Integrated Math III

Number and Quantity

Quantities

A. Reason quantitatively and use units to understand problems. M3.N.Q.A

1. Use units as a way to understand real-world problems. M3.N.Q.A.1

a. Choose and interpret the scale and the origin in graphs and data displays. M3.N.Q.A.1.A

b. Use appropriate quantities in formulas, converting units as necessary. M3.N.Q.A.1.B

c. Define and justify appropriate quantities within a context for the purpose of modeling. M3.N.Q.A.1.C

d. Choose an appropriate level of accuracy when reporting quantities. M3.N.Q.A.1.D

Algebra

Seeing Structure in Expressions

A. Interpret the structure of expressions. M3.A.SSE.A

1. Interpret expressions that represent a quantity in terms of its context. M3.A.SSE.A.1
    a. Interpret parts of an expression, such as terms, factors, and coefficients. M3.A.SSE.A.1.A
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. M3.A.SSE.A.1.B

Arithmetic with Polynomials and Rational Expressions

A. Understand the relationship between zeros and factors of polynomials. M3.A.APR.A

1. Know and apply the Factor Theorem: For a polynomial p(x) and a number a, p(a) = 0 if and only if (x – a) is a factor of p(x). M3.A.APR.A.1
2. Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial. M3.A.APR.A.2

Creating Equations

A. Create equations that describe numbers or relationships. M3.A.CED.A

1. Create equations and inequalities in one variable and use them to solve problems in a real-world context. M3.A.CED.A.1
2. Create equations and inequalities in two variables to represent relationships between quantities and use them to solve problems in a real-world context. Graph equations and inequalities with two variables on coordinate axes with labels and scales, and use the graphs to make predictions. M3.A.CED.A.2
3. Rearrange formulas to isolate a quantity of interest using algebraic reasoning. M3.A.CED.A.3

Reasoning with Equations and Inequalities

A. Understand solving equations as a process of reasoning and explain the reasoning. M3.A.REI.A

1. Understand solving equations as a process of reasoning and explain the reasoning. Construct a viable argument to justify a solution method. M3.A.REI.A.1
2. Solve radical equations in one variable and identify extraneous solutions when they exist. M3.A.REI.A.2

Functions

Interpreting Functions

A. Understand the concept of a function and use function notation. M3.F.IF.A

1. Use function notation. M3.F.IF.A.1

   a. Use function notation to evaluate functions for inputs in their domains, including functions of two variables. M3.F.IF.A.1.A

   b. Interpret statements that use function notation in terms of a context. M3.F.IF.A.1.B

2. Understand geometric formulas as functions. M3.F.IF.A.2

B. Interpret functions that arise in applications in terms of the context. M3.F.IF.B

3. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. M3.F.IF.B.3

4. Calculate and interpret the average rate of change of a function (presented algebraically or as a table) over a specified interval. Estimate and interpret the rate of change from a graph. M3.F.IF.B.4

C. Analyze functions using different representations. M3.F.IF.C

5. Graph functions expressed algebraically and show key features of the graph by hand and using technology. M3.F.IF.C.5

6. Compare properties of functions represented algebraically, graphically, numerically in tables, or by verbal descriptions. M3.F.IF.C.6

   a. Compare properties of two different functions. Functions may be of different types and/or represented in different ways. M3.F.IF.C.6.A

   b. Compare properties of the same function on two different intervals or represented in two different ways. M3.F.IF.C.6.B

Building Functions

A. Build new functions from existing functions. M3.F.BF.A

1. Build a function that describes a relationship between two quantities. M3.F.BF.A.1

   a. Combine standard function types using composition. M3.F.BF.A.1.A

2. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given graphs. M3.F.BF.A.2

3. Find the inverse of a function. M3.F.BF.A.3

   a. Determine whether a function is one-to-one. M3.F.BF.A.3.A

   b. Find the inverse of a function on an appropriate domain. M3.F.BF.A.3.B

   c. Given an invertible function on an appropriate domain, identify the domain of the inverse function. M3.F.BF.A.3.C

Linear, Quadratic, and Exponential Models

A. Construct and compare linear, quadratic, and exponential models and solve problems. M3.F.LE.A

   1. Know that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or cubically. M3.F.LE.A.1

   2. Know the relationship between exponential functions and logarithmic functions. M3.F.LE.A.2

      a. Solve exponential equations using a variety of strategies, including logarithms. M3.F.LE.A.2.A

      b. Understand that a logarithm is the solution to ab^{ct} = d, where a, b, c, and d are numbers. M3.F.LE.A.2.B

      c. Evaluate logarithms using technology. M3.F.LE.A.2.C

Geometry

Circles

A. Find areas of sectors of circles. M3.G.C.A

1. Use proportional relationships between the area of a circle and the area of a sector within the circle to solve problems and represent solutions in a real-world context. M3.G.C.A.1

Similarity, Right Triangles, and Trigonometry

A. Define trigonometric ratios and solve problems involving triangles. M3.G.SRT.A

1. Use side ratios in right triangles to define trigonometric ratios. M3.G.SRT.A.1
    a. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. M3.G.SRT.A.1.A
    b. Explain and use the relationship between the sine and cosine of complementary angles. M3.G.SRT.A.1.B
2. Solve triangles. M3.G.SRT.A.2
    a. Know and use the Pythagorean Theorem and trigonometric ratios (sine, cosine, tangent, and their inverses) to solve right triangles in a real-world context. M3.G.SRT.A.2.A
    b. Know and use relationships within special right triangles to solve problems in a real-world context. M3.G.SRT.A.2.B
    c. Use the Law of Sines and Law of Cosines to solve non-right triangles in a real-world context. M3.G.SRT.A.2.C

Modeling with Geometry

A. Apply geometric concepts in modeling situations. M3.G.MG.A

1. Use geometric shapes, their measures, and their properties to model objects found in a real-world context for the purpose of approximating solutions to problems. M3.G.MG.A.1

Geometric Measurement and Dimension

A. Explain volume and surface area formulas and use them to solve problems. M3.G.GMD.A

1. Understand and explain the formulas for the volume and surface area of a cylinder, cone, prism, and pyramid. M3.G.GMD.A.1
2. Use volume and surface area formulas for cylinders, cones, prisms, pyramids, and spheres to solve problems in a real-world context. M3.G.GMD.A.2

Statistics and Probability

Interpreting Categorical and Quantitative Data

A. Summarize, represent, and interpret data on a single count or measurement variable. M3.S.ID.A

1. Use measures of center to solve real-world and mathematical problems. M3.S.ID.A.1
2. Use statistics appropriate to the shape of the data distribution to compare center (mean, median, and/or mode) and spread (range, interquartile range, and standard deviation) of two or more different data sets. M3.S.ID.A.2
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points. M3.S.ID.A.3
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages using the Empirical Rule. M3.S.ID.A.4
5. Compute, interpret, and compare z-scores for normally distributed data in a real-world context. M3.S.ID.A.5

B. Summarize, represent, and interpret data on two categorical and quantitative variables. M3.S.ID.B

6. Represent data from two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. M3.S.ID.B.6

Making Inferences and Justifying Conclusions

A. Make inferences and justify conclusions from sample surveys, experiments, and observational studies. M3.S.IC.A

1. Recognize the purposes of and differences among sample surveys, experiments, and observational studies. M3.S.IC.A.1
2. Identify potential sources of bias in statistical studies. M3.S.IC.A.2
3. Distinguish between a statistic and a parameter. Evaluate reports based on data and recognize when poor conclusions are drawn from well-collected data. M3.S.IC.A.3

Conditional Probability and the Rules of Probability

A. Understand independence and conditional probability and use them to create visual representations of data. M3.S.CP.A

1. Use set notation to represent contextual situations. M3.S.CP.A.1
   a. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or", "and", "not"). M3.S.CP.A.1.A
   b. Flexibly move between visual models (Venn diagrams, frequency tables, etc.) and set notation. M3.S.CP.A.1.B

2. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. Categorize events as independent or dependent. M3.S.CP.A.2

B. Understand and apply basic concepts of probability. M3.S.CP.B

3. Apply statistical counting techniques. M3.S.CP.B.3

   a. Use the Fundamental Counting Principle to compute probabilities of compound events and solve problems. M3.S.CP.B.3.A

   b. Use permutations and combinations to compute probabilities of compound events and solve problems. M3.S.CP.B.3.B

4. Use the Law of Large Numbers to assess the validity of a statistical claim. M3.S.CP.B.4

C. Use the rules of probability to compute probabilities of compound events in a uniform probability model. M3.S.CP.C

5. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the given context. M3.S.CP.C.5

6. Understand and apply the Addition Rule. M3.S.CP.C.6

   a. Explain the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B) in terms of visual models (Venn diagrams, frequency tables, etc.). M3.S.CP.C.6.A

   b. Apply the Addition Rule to solve problems and interpret the answer in terms of the given context. M3.S.CP.C.6.B

D. Apply geometric concepts to situations involving probability. M3.S.CP.D

7. Calculate probabilities using geometric figures. M3.S.CP.D.7

Mathematical Reasoning for Decision Making

Number and Quantity

Financial Mathematics

A. Use financial mathematics to make personal financial decisions. MR.N.NQ.A

1. Define common terms associated with finance (such as interest, compound interest, annuities, retirement funds, amortizations, future value, and present value) and know how each term is related to personal finance. MR.N.NQ.A.1
2. Calculate compound interest within the context of personal finance (such as credit card debt, home/car purchase, personal loans, and amortization schedules) and use the results to make decisions (for example, determine which home financing option is best). MR.N.NQ.A.2
3. Calculate net pay using gross pay (weekly, biweekly, monthly, or annual) and both fixed and variable deductions (such as withholding tax, Social Security tax, insurance costs, retirement investments and other contributory benefits). MR.N.NQ.A.3
4. Access and use published data (such as cost of city or state utilities, housing, city or state taxes, meals, and other costs of living) to estimate and compare monthly living expenses based on location, identified needs, and personal preferences or desired lifestyles. MR.N.NQ.A.4
5. Access and use published data (such as average life expectancy based on location and/or health issues, investment data, retirement funds, and annuity data) to calculate and compare retirement investments (such as total savings and monthly payouts) based on projected income. MR.N.NQ.A.5
6. Access and use published data to create depreciation schedules and analyze the depreciation of various assets (such as cars, business equipment, and store fixtures). MR.N.NQ.A.6
7. Access and use published data to calculate income tax based on projected gross annual income, returns on investments, tax deductions and tax credits, and other factors that affect calculations. MR.N.NQ.A.7
8. Develop a personal mid-term (three to five years) financial plan based on anticipated income, projected living expenses, projected retirement or other savings, and other factors that affect personal finances. MR.N.NQ.A.8

B. Use financial mathematics to make business decisions. MR.N.NQ.B

9. Compare the components of a small business plan to the components of a personal financial plan (i.e., identify components that are common to both plans and components that are unique to a small business plan). MR.N.NQ.B.9
10. Define common terms associated with business finance (such as assets, liabilities, revenue, expenses, net profit, net loss, profit margin, and return on investment) and know how each term is related to business finance. MR.N.NQ.B.10
11. Access and use published data to develop a three-year financial plan for starting and running a small business (including projected income and projected fixed and variable costs such as licenses, rent and utilities, city and state taxes, cost of goods sold, etc.). MR.N.NQ.B.11

Algebra

Linear Programming

A. Use linear programming techniques to solve real-world problems. MR.A.LP.A

1. Read, interpret, and solve linear programming problems graphically and by computational methods. MR.A.LP.A.1

B. Solve real-world optimization problems. MR.A.LP.B

2. Use linear programming to solve optimization problems (for example, optimizing profit for a small business). MR.A.LP.B.2
3. Interpret the meaning of the maximum or minimum value in terms of the objective function. MR.A.LP.B.3

Data Analysis, Statistics, and Probability

Organize and Interpret Data

A. Analyze data from multiple viewpoints and perspectives. MR.D.ID.A

1. Organize, analyze, and interpret data for problem solving (for example, compare data related to costs of living; analyze survey data; provide a circle graph that demonstrates the percentages of income that support various expenses). MR.D.ID.A.1
2. Determine whether a set of data supports a given assertion (for example, whether a data set collected on Tennessee residents can be generalized to support an assertion about all Americans; whether a data set supports, or is large enough to support, the validity of a claim). MR.D.ID.A.2
3. Develop facility with representations of a data set and explain why some representations are more accurate or relevant than others in a given context. MR.D.ID.A.3
4. Interpret and use measures of central tendency and spread to solve problems and make informed decisions. MR.D.ID.A.4
5. Calculate expected value in real-world situations (such as lottery return on investment, expected value of each possession in sports, and expected payoff in a game of chance). MR.D.ID.A.5
6. Evaluate and compare two investments or strategies where one investment or strategy is safer but has lower expected value. Include large and small investments and situations with serious consequences. MR.D.ID.A.6
7. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. Evaluate strategies and make decisions based on expected values (for example, whether a team should pursue a higher-scoring option with a smaller probability of success or a lower-scoring option with a higher probability of success; whether a homeowner should file a small insurance claim given the probability that the monthly cost of insurance will rise as a result). MR.D.ID.A.7

Normal Probability Distribution

A. Work with the normal distribution in real-world situations. MR.D.ND.A

1. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. MR.D.ND.A.1

B. Work with the confidence intervals in real-world situations. MR.D.ND.B

2. Understand and interpret confidence levels and confidence intervals (for example, use the weights of randomly sampled boxes of cereal compared to the expected tolerances to determine whether the machinery is operating properly). MR.D.ND.B.2

Geometry and Measurement

Geometric Measurement

A. Understand the role of precision in measurement. MR.G.GMD.A

1. Use standard units (metric and non-metric) to accurately measure objects to within 0.1 of the unit used. MR.G.GMD.A.1
2. Use precise measurements (within 0.1 of the unit used) to calculate area, surface area, and volume/capacity (emphasize common two-and three-dimensional shapes). MR.G.GMD.A.2
3. Understand and explain the effects that an error in measurement will have on a calculation that uses the erroneous measurement (for example, whether an error of 0.1 unit in length affects the calculated vs. actual measurement of the volume of an object, and whether that error is compounded by errors in other measurements used in the calculation). MR.G.GMD.A.3

B. Accurately use standard and nonstandard units in measurement. MR.G.GMD.B

4. Use standard units of measure to develop accurately estimated measurements of commonly available non-standard instruments of measurement (for example, establish the length of hand span in inches or centimeters; length of arm span or stride length in feet or yards; the area of a floor tile in square inches or square feet; the volume of a gallon of milk or a water bottle or a soda can in cubic inches or cubic centimeters, etc.). MR.G.GMD.B.4
5. Understand and explain the consequences of relying on nonstandard units of measure (for example, explain why paper clip length or pencil length are not standard units of measure and how failing to use mutually agreed upon units can lead to erroneous assumptions, calculations, or conclusions). MR.G.GMD.B.5
6. Use the established dimensions of common non-standard measuring instruments to estimate other measurements using standard units to a given tolerance (for example, use stride length to estimate the length of a hallway to within 10% of the actual length in feet; use the estimated volume of a finger in cubic centimeters to estimate the amount of liquid in a glass). MR.G.GMD.B.6

C. Accurately use standard and nonstandard units in measurement. MR.G.GMD.C

7. Estimate the area, surface area, volume, or capacity of an object using the established dimensions of common non-standard measuring instruments to determine measurements in standard units with and without using technology (for example, use the number of floor tiles along a wall to estimate the area of the floor of a room and use the height of a person to estimate the height of the room, then find the volume of the room based on those estimations; use the size of a milk jug to estimate the number of gallons in a tank of water). MR.G.GMD.C.7
8. Estimate the amount of error in a calculation that is based on using established dimensions of common non-standard measuring instruments (for example, if a person's stride length is 30 inches plus/minus 2 inches, and the

person uses stride length to measure the length and width of a plot of land, determine the estimated error in calculating the area of the plot of land). MR.G.GMD.C.8

9. Understand and use unit conversions in estimations involving both standard and non-standard units (for example, determine how many boxes of flooring will be needed to cover a floor of given dimensions if 10% waste is assumed; how many gallons of paint will be needed to paint a room of a given size; how many bags of fertilizer will be needed to fertilize a yard of a given size). MR.G.GMD.C.9

10. Discuss the various examples and consequences of innumeracy; consider poor estimation, improper experimental design, inappropriate comparisons, and scientific notation comparisons. MR.G.GMD.C.10

Statistics

1. **Sampling and Data** S.1

   a. Understand the investigative process of statistics and differentiate between descriptive and inferential statistics. S.1.A

   b. Differentiate between a population and a sample. S.1.B

   c. Construct a simple random sample. S.1.C

   d. Understand the differences between stratified sampling, cluster sampling, systematic sampling, and convenience sampling. S.1.D

   e. Determine when samples of convenience are acceptable and how sampling bias and error can occur. S.1.E

   f. Identify and classify data as either qualitative or quantitative and classify quantitative data as either discrete or continuous data. S.1.F

   g. Display and interpret qualitative data with graphs: pie graphs, bar graphs, and pareto charts. S.1.G

   h. Differentiate between levels of measurement: nominal, ordinal, interval, and ratio. S.1.H

   i. Create a frequency distribution from a list of quantitative and/or qualitative data. S.1.I

   j. Calculate relative frequencies and cumulative frequencies using a frequency distribution table. S.1.J

   k. Understand differences between a designed experiment and an observational study. S.1.K

   l. Differentiate between the types of variables used in a designed experiment. S.1.L

   m. Understand different methods used in an experiment to isolate effects of the explanatory variable. S.1.M

2. **Descriptive Statistics** S.2

   a. Display and interpret graphs using quantitative data including stem-and-leaf plots, line graphs, and box plots. S.2.A

   b. Construct a histogram from a frequency distribution table. S.2.B

   c. Interpret data using histograms and time series graphs. S.2.C

   d. Analyze a frequency distribution table and determine the sample size, class width and class midpoints. S.2.D

   e. Recognize, describe, and calculate the measures of locations of data: quartiles, median, five number summary, interquartile range outliers, upper and lower fences, and percentiles. S.2.E

   f. Distinguish between a parameter and a statistic. S.2.F

   g. Calculate and differentiate between different measures of center: mean, median, and mode. S.2.G

   h. Calculate the mean of a frequency distribution: GPA and weighted grade. S.2.H

   i. Interpret the shape of the distribution from a graph: normal/symmetric, skewed, or uniform. S.2.I

   j. Calculate and differentiate between different measures of spread: range, variance, and standard deviation. S.2.J

   k. Determine if a data value is unusual based on standard deviations, $\mu \pm 2\sigma$. S.2.K

3. **Probability** S.3

   a. Understand and use terminology and symbols of probability. S.3.A

   b. List the elements of events and the sample space from an experiment. S.3.B

   c. Understand the concept of randomness: flipping a coin, rolling a die, and drawing a card from a standard 52 card deck. S.3.C

   d. Differentiate between and calculate different types of probabilities: empirical and theoretical. S.3.D

   e. Explain the Law of Large Numbers. S.3.E

   f. Calculate and interpret probabilities using the complement rule, addition rule, and multiplication rule. S.3.F

   g. Differentiate between and calculate probabilities for different types of events: independent, dependent, with or without replacement, conditional, and mutually exclusive. S.3.G

   h. Use Venn diagrams and lists to solve probability problems when appropriate. S.3.H

4. **Discrete Random Variables** S.4

   a. Identify the random variable in a probability experiment. S.4.A

   b. Recognize and understand discrete probability distribution functions. S.4.B

   c. Create a probability distribution for the values of a discrete random variable. S.4.C

   d. Use a probability function to determine probabilities associated with a discrete random variable. S.4.D

   e. Calculate and interpret the mean (expected value), variance, and standard deviation for discrete random variables and binomial probability distributions. S.4.E

   f. Determine when a probability distribution should be classified as a discrete binomial probability distribution, and calculate probabilities associated with such a distribution. S.4.F

5. **Continuous Random Variables and the Normal Distribution** S.5

   a. Recognize and understand continuous probability density functions. S.5.A

   b. Use a probability density curve to describe a population, including a normal population. S.5.B

   c. Calculate and interpret the area under a probability density curve. S.5.C

   d. Calculate and interpret a z-score, understanding the concept of "standardizing" data. S.5.D

   e. Calculate and interpret z-scores using the Empirical Rule, understanding the general properties of the normal distribution: 100% is the total area under the curve, exactly 50% is to the left and right of the mean, and it is perfectly symmetric about the mean. S.5.E

   f. Use technology to calculate the area under the curve for any normal distribution model: left, right, and between. S.5.F

   g. Use technology to calculate percentiles, quartiles, and other numerical values of X for a specified area under a normal curve, including unusual values ($P(X) < 5\%$ and $\mu \pm 2\sigma$). S.5.G

6. **Central Limit Theorem** S.6

   a. Recognize the characteristics of the mean of sample means taken from different types of populations: normal and non-normal. S.6.A

   b. Calculate the mean of sample means taken from different types of populations: normal and non-normal. S.6.B

   c. Describe how the means of samples calculated from a non-normal population might be distributed. S.6.C

   d. Apply the Central Limit Theorem to normal and non-normal populations and compute probabilities of a sample mean. S.6.D

   e. Determine whether the Central Limit Theorem can be used for a given situation. S.6.E

   f. Assess the impact of sample size on sampling variability. S.6.F

7. **Confidence Intervals** S.7

   a. Read and write confidence intervals using two different forms: point estimate plus/or minus margin of error (error bound) and interval notation. S.7.A

   b. Calculate and interpret confidence intervals for estimating a population mean and a population proportion. S.7.B

   c. Calculate the margin of error (error bound) using sample statistics. S.7.C

   d. Predict if a confidence interval will become wider or narrower given larger or smaller sample sizes as well as higher or lower confidence levels. S.7.D

   e. Find the point estimate and margin of error (error bound) when given a confidence interval. S.7.E

   f. Estimate the sample size necessary to estimate a population mean. S.7.F

   g. Recognize the difference between the sample mean, and the population mean, μ, as well as the difference between the sample standard deviation, s, and standard error of the mean, s/√n. S.7.G

   h. Find critical values for Z_{α/2} and t_{α/2} given a value of α and degrees of freedom. S.7.H

   i. Estimate the sample size necessary to estimate a population proportion. S.7.I

8. **Hypothesis Testing** S.8
    a. Determine the appropriate null and alternative hypotheses when presented with a problem. S.8.A
    b. Differentiate between Type I and Type II errors. S.8.B
    c. Understand and list the assumptions needed to conduct z-tests and t-tests. S.8.C
    d. Determine whether to reject or fail to reject the null hypothesis using the p-value method. S.8.D
    e. Determine if a test is left-tailed, right-tailed, or two-tailed. S.8.E
    f. Differentiate between independent group and matched pair sampling. S.8.F
    g. Calculate test statistics and p-values for hypotheses tests: single proportion, single mean, and difference between two means. S.8.G
    h. Conduct hypotheses tests for a single proportion and a single mean. S.8.H
    i. Test hypotheses regarding the difference of two independent means (assume the variances are not pooled). S.8.I
    j. Draw conclusions and make inferences about claims based on hypotheses tests. S.8.J

9. **Regression Correlation** S.9
    a. Differentiate between the independent (explanatory variable, x) and the dependent (response variable, y) in a bivariate data set. S.9.A
    b. Create a scatter plot and determine the type of relationship that exists between two variables: positive or negative correlation and weak or strong correlation. S.9.B
    c. Calculate and interpret the correlation coefficient using technology. S.9.C
    d. Calculate the line of best fit and interpret the coefficient of determination. S.9.D
    e. Use the line of best fit to make conclusions about the relationship between two variables, understanding correlation does not imply causation. S.9.E
    f. Calculate a residual using the line of best fit. S.9.F
    g. Use the p-value to determine if a line of best fit is statistically significant. S.9.G
    h. For a given value of x, find the appropriate estimated value of y. S.9.H
    i. Distinguish between interpolated and extrapolated values and explain why interpolated values are more reliable. S.9.I
    j. Perform a residual analysis to check assumptions of regression. S.9.J

Precalculus

Number and Quantity

Number Expressions

A. Represent, interpret, compare, and simplify number expressions. P.N.NE.A

1. Use the laws of exponents and logarithms to expand or collect terms in expressions; simplify expressions or modify them in order to analyze them or compare them. P.N.NE.A.1
2. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. P.N.NE.A.2
3. Classify real numbers and order real numbers that include transcendental expressions, including roots and fractions of π and e. P.N.NE.A.3
4. Simplify complex radical and rational expressions; discuss and display understanding that rational numbers are dense in the real numbers and the integers are not. P.N.NE.A.4
5. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. P.N.NE.A.5

The Complex Number System

A. Perform complex number arithmetic and understand the representation on the complex plane. P.N.CN.A

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real. P.N.CN.A.1
2. Perform arithmetic operations with complex numbers expressing answers in the form a + bi. P.N.CN.A.2
3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. P.N.CN.A.3
4. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. P.N.CN.A.4
5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation (for example, $(-1 + 3i)^3 = 8$ because $(-1 + 3i)$ has modulus 2 and argument 120°). P.N.CN.A.5
6. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. P.N.CN.A.6

B. Use complex numbers in polynomial identities and equations. P.N.CN.B

7. Extend polynomial identities to the complex numbers (for example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$). P.N.CN.B.7

8. Solve quadratic equations with real coefficients that have complex solutions. P.N.CN.B.8

9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. P.N.CN.B.9

Vector and Matrix Quantities

A. Represent and model with vector quantities. P.N.VM.A

1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, . P.N.VM.A.1

2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. P.N.VM.A.2

3. Solve problems involving velocity and other quantities that can be represented by vectors. P.N.VM.A.3

B. Understand the graphic representation of vectors and vector arithmetic. P.N.VM.B

4. Add and subtract vectors. P.N.VM.B.4

   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. P.N.VM.B.4.A

   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. P.N.VM.B.4.B

   c. Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. P.N.VM.B.4.C

5. Multiply a vector by a scalar. P.N.VM.B.5

   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise (e.g., as c(v_x, v_y) = (cv_x, cv_y). P.N.VM.B.5.A

   b. Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when |c|v ≠ 0, the direction of cv is either along v (for c > 0) or against v (for c < 0). P.N.VM.B.5.B

6. Calculate and interpret the dot product of two vectors. P.N.VM.B.6

C. Perform operations on matrices and use matrices in applications. P.N.VM.C

7. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. P.N.VM.C.7

8. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. P.N.VM.C.8

9. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. P.N.VM.C.9

10. Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. P.N.VM.C.10

Algebra

Sequences and Series

A. Understand and use sequences and series. P.A.S.A

1. Demonstrate an understanding of sequences by representing them recursively and explicitly. P.A.S.A.1
2. Use sigma notation to represent a series; expand and collect expressions in both finite and infinite settings. P.A.S.A.2
3. Derive and use the formulas for the general term and summation of finite or infinite arithmetic and geometric series, if they exist. P.A.S.A.3
   a. Determine whether a given arithmetic or geometric series converges or diverges. P.A.S.A.3.A
   b. Find the sum of a given geometric series (both infinite and finite). P.A.S.A.3.B
   c. Find the sum of a finite arithmetic series. P.A.S.A.3.C
4. Understand that series represent the approximation of a number when truncated; estimate truncation error in specific examples. P.A.S.A.4
5. Know and apply the Binomial Theorem for the expansion of (x + y) ⁿ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined, for example, by Pascal's Triangle. P.A.S.A.5

Reasoning with Equations and Inequalities

A. Solve systems of equations and nonlinear inequalities. P.A.REI.A

1. Represent a system of linear equations as a single matrix equation in a vector variable. P.A.REI.A.1
2. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). P.A.REI.A.2
3. Solve rational and radical equations in one variable, and identify extraneous solutions when they exist. P.A.REI.A.3
4. Solve nonlinear inequalities (quadratic, trigonometric, conic, exponential, logarithmic, and rational) by graphing (solutions in interval notation if one-variable), by hand and with appropriate technology. P.A.REI.A.4
5. Solve systems of nonlinear inequalities by graphing. P.A.REI.A.5

Parametric Equations

A. Describe and use parametric equations. P.A.PE.A

1. Graph curves parametrically (by hand and with appropriate technology). P.A.PE.A.1
2. Eliminate parameters by rewriting parametric equations as a single equation. P.A.PE.A.2

Conic Sections

A. Understand the properties of conic sections and model real-world phenomena. P.A.C.A

1. Display all of the conic sections as portions of a cone. P.A.C.A.1
2. Know and write the equation of a circle of given center and radius using the Pythagorean Theorem. P.A.C.A.2
3. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. P.A.C.A.3
4. From an equation in standard form, graph the appropriate conic section: ellipses, hyperbolas, circles, and parabolas. Demonstrate an understanding of the relationship between their standard algebraic form and the graphical characteristics. P.A.C.A.4
5. Transform equations of conic sections to convert between general and standard form. P.A.C.A.5

Functions

Building Functions

A. Build new functions from existing functions. P.F.BF.A

1. Understand how the algebraic properties of an equation transform the geometric properties of its graph (for example, given a function, describe the transformation of the graph resulting from the manipulation of the algebraic properties of the equation such as translations, stretches, reflections, and changes in periodicity and amplitude). P.F.BF.A.1
2. Develop an understanding of functions as elements that can be operated upon to get new functions: addition, subtraction, multiplication, division, and composition of functions. P.F.BF.A.2
3. Compose functions (for example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time). P.F.BF.A.3
4. Construct the difference quotient for a given function and simplify the resulting expression. P.F.BF.A.4
5. Find inverse functions (including exponential, logarithmic, and trigonometric). P.F.BF.A.5
    a. Calculate the inverse of a function, f (x), with respect to each of the functional operations; in other words, the additive inverse, − f (x), the multiplicative inverse, 1/f(x), and the inverse with respect to composition, f ^{−1}(x). Understand the algebraic and graphical implications of each type. P.F.BF.A.5.A
    b. Verify by composition that one function is the inverse of another. P.F.BF.A.5.B
    c. Read values of an inverse function from a graph or a table, given that the function has an inverse. P.F.BF.A.5.C
    d. Recognize a function is invertible if and only if it is one-to-one. Produce an invertible function from a non-invertible function by restricting the domain. P.F.BF.A.5.D
6. Explain why the graph of a function and its inverse are reflections of one another over the line y = x. P.F.BF.A.6

Interpreting Functions

A. Analyze functions using different representations. P.F.IF.A

1. Determine whether a function is even, odd, or neither. P.F.IF.A.1
2. Analyze qualities of exponential, polynomial, logarithmic, trigonometric, and rational functions and solve real-world problems that can be modeled with these functions (by hand and with appropriate technology). P.F.IF.A.2
3. Identify the real zeros of a function and explain the relationship between the real zeros and the x-intercepts of the graph of a function (exponential,

polynomial, logarithmic, trigonometric, and rational). P.F.IF.A.3

4. Identify characteristics of graphs based on a set of conditions or on a general equation such as $y = ax^2 + c$. P.F.IF.A.4
5. Visually locate critical points on the graphs of functions and determine if each critical point is a minimum, a maximum, or point of inflection. Describe intervals where the function is increasing or decreasing and where different types of concavity occur. P.F.IF.A.5
6. Graph rational functions, identifying zeros, asymptotes (including slant), and holes (when suitable factorizations are available) and showing end behavior. P.F.IF.A.6
7. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers (for example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$). P.F.IF.A.7

Trigonometric Functions

A. Extend the domain of trigonometric functions using the unit circle. P.F.TF.A

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. P.F.TF.A.1
2. Convert from radians to degrees and from degrees to radians. P.F.TF.A.2
3. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and explain how to use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number. P.F.TF.A.3
4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. P.F.TF.A.4
5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. P.F.TF.A.5

Graphing Trigonometric Functions

A. Model periodic phenomena with trigonometric functions. P.F.GT.A

1. Interpret transformations of trigonometric functions. P.F.GT.A.1
2. Determine the difference made by choice of units for angle measurement when graphing a trigonometric function. P.F.GT.A.2
3. Graph the six trigonometric functions and identify characteristics such as period, amplitude, phase shift, and asymptotes. P.F.GT.A.3
4. Find values of inverse trigonometric expressions (including compositions), applying appropriate domain and range restrictions. P.F.GT.A.4
5. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. P.F.GT.A.5
6. Determine the appropriate domain and corresponding range for each of the inverse trigonometric functions. P.F.GT.A.6

7. Graph the inverse trigonometric functions and identify their key characteristics. P.F.GT.A.7
8. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context. P.F.GT.A.8

Geometry

Applied Trigonometry

A. Use trigonometry to solve problems. P.G.AT.A

1. Use the definitions of the six trigonometric ratios as ratios of sides in a right triangle to solve problems about lengths of sides and measures of angles. P.G.AT.A.1
2. Derive the formula A = ½ ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. P.G.AT.A.2
3. Derive and apply the formulas for the area of sector of a circle. P.G.AT.A.3
4. Calculate the arc length of a circle subtended by a central angle. P.G.AT.A.4
5. Prove the Laws of Sines and Cosines and use them to solve problems. P.G.AT.A.5
6. Understand and apply the Law of Sines (including the ambiguous case) and the Law of Cosines to find unknown measurements in right and non-right triangles (such as surveying problems and resultant forces). P.G.AT.A.6

Trigonometric Identities

A. Apply trigonometric identities to rewrite expressions and solve equations. P.G.TI.A

1. Apply trigonometric identities to verify identities and solve equations. Identities include: Pythagorean, reciprocal, quotient, sum/difference, double-angle, and half-angle. P.G.TI.A.1
2. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. P.G.TI.A.2

Polar Coordinates

A. Use polar coordinates. P.G.PC.A

1. Graph functions in polar coordinates. P.G.PC.A.1
2. Convert between rectangular and polar coordinates. P.G.PC.A.2
3. Represent situations and solve problems involving polar coordinates. P.G.PC.A.3

Statistics and Probability

Model with Data

A. Model data using regressions equations. P.S.MD.A

1. Create scatter plots, analyze patterns, and describe relationships for bivariate data (linear, polynomial, trigonometric, or exponential) to model real-world phenomena and to make predictions. P.S.MD.A.1
2. Determine a regression equation to model a set of bivariate data. Justify why this equation best fits the data. P.S.MD.A.2
3. Use a regression equation, modeling bivariate data, to make predictions. Identify possible considerations regarding the accuracy of predictions when interpolating or extrapolating. P.S.MD.A.3

Calculus

Functions, Graphs, and Limits

Limits of Functions

A. Understand the concept of the limit of a function. C.F.LF.A

1. Calculate limits (including limits at infinity) using algebra. C.F.LF.A.1
2. Estimate limits of functions (including one-sided limits) from graphs or tables of data. Apply the definition of a limit to a variety of functions, including piecewise functions. C.F.LF.A.2
3. Draw a sketch that illustrates the definition of the limit; develop multiple real-world scenarios that illustrate the definition of the limit. C.F.LF.A.3

Behavior of Functions

A. Describe the asymptotic and unbounded behavior of functions. C.F.BF.A

1. Describe asymptotic behavior (analytically and graphically) in terms of infinite limits and limits at infinity. C.F.BF.A.1
2. Discuss the various types of end behavior of functions; identify prototypical functions for each type of end behavior. C.F.BF.A.2

Continuity

A. Develop an understanding of understanding of continuity as a property of functions C.F.C.A

1. Define continuity at a point using limits; define continuous functions. C.F.C.A.1
2. Determine whether a given function is continuous at a specific point. C.F.C.A.2
3. Determine and define different types of discontinuity (point, jump, infinite) in terms of limits. C.F.C.A.3
4. Apply the Intermediate Value Theorem and Extreme Value Theorem to continuous functions. C.F.C.A.4

Derivatives

Understand the Concept of the Derivative

A. Demonstrate an understanding of the derivative. C.D.CD.A

1. Represent and interpret the derivative of a function graphically, numerically, and analytically. C.D.CD.A.1
2. Interpret the derivative as an instantaneous rate of change. C.D.CD.A.2
3. Define the derivative as the limit of the difference quotient; illustrate with the sketch of a graph. C.D.CD.A.3
4. Demonstrate the relationship between differentiability and continuity. C.D.CD.A.4

B. Understand the derivative at a point. C.D.CD.B

5. Interpret the derivative as the slope of a curve (which could be a line) at a point, including points at which there are vertical tangents and points at which there are no tangents (i.e., where a function is not locally linear). C.D.CD.B.5
6. Approximate both the instantaneous rate of change and the average rate of change given a graph or table of values. C.D.CD.B.6
7. Write the equation of the line tangent to a curve at a given point. C.D.CD.B.7
8. Apply the Mean Value Theorem. C.D.CD.B.8
9. Understand Rolle's Theorem as a special case of the Mean Value Theorem. C.D.CD.B.9

Computing and Applying Derivatives

A. Apply differentiation techniques. C.D.AD.A

1. Describe in detail how the basic derivative rules are used to differentiate a function; discuss the difference between using the limit definition of the derivative and using the derivative rules. C.D.AD.A.1
2. Calculate the derivative of basic functions (power, exponential, logarithmic, and trigonometric). C.D.AD.A.2
3. Calculate the derivatives of sums, products, and quotients of basic functions. C.D.AD.A.3
4. Apply the chain rule to find the derivative of a composite function. C.D.AD.A.4
5. Implicitly differentiate an equation in two or more variables C.D.AD.A.5
6. Use implicit differentiation to find the derivative of the inverse of a function. C.D.AD.A.6

B. Use first and second derivatives to analyze a function. C.D.AD.B

7. Relate the increasing and decreasing behavior of f to the sign of f' both analytically and graphically. C.D.AD.B.7

8. Use the first derivative to find extrema (local/relative and global/absolute). C.D.AD.B.8
9. Analytically locate the intervals on which a function is increasing, decreasing, or neither. C.D.AD.B.9
10. Relate the concavity of f to the sign of ff" both analytically and graphically. C.D.AD.B.10
11. Use the second derivative to find points of inflection as points where concavity changes. C.D.AD.B.11
12. Analytically locate intervals on which a function is concave up, concave down, or neither. C.D.AD.B.12
13. Relate corresponding characteristics of the graphs of f , f', and f". C.D.AD.B.13
14. Translate verbal descriptions into equations involving derivatives and vice versa. C.D.AD.B.14

C. Apply derivatives to solve problems. C.D.AD.C

15. Model rates of change, including related rates problems. In each case, include a discussion of units. C.D.AD.C.15
16. Solve optimization problems to find a desired maximum or minimum value. C.D.AD.C.16
17. Use differentiation to solve problems involving velocity, speed, and acceleration. C.D.AD.C.17
18. Use tangent lines to approximate function values and changes in function values when inputs change (linearization). C.D.AD.C.18

Integrals

Understanding Integrals

A. Demonstrate understanding of a definite integral. C.I.UI.A

1. Define the definite integral as the limit of Riemann sums and as the net accumulation of change. C.I.UI.A.1
2. Write a Riemann sum that represents the definition of a definite integral. C.I.UI.A.2
3. Use Riemann sums (left, right, and midpoint evaluation points) and trapezoid sums to approximate definite integrals of functions represented graphically, numerically, and by tables of values. C.I.UI.A.3

B. Understand and apply the Fundamental Theorem of Calculus. C.I.UI.B

4. Recognize differentiation and antidifferentiation as inverse operations. C.I.UI.B.4
5. Evaluate definite integrals using the Fundamental Theorem of Calculus. C.I.UI.B.5
6. Use the Fundamental Theorem of Calculus to represent a particular antiderivative of a function and to understand when the antiderivative so represented is continuous and differentiable. C.I.UI.B.6
7. Apply basic properties of definite integrals (e.g. additive, constant multiple, translations). C.I.UI.B.7

Calculate and Apply Integrals

A. Apply techniques of antidifferentiation. C.I.AI.A

1. Find antiderivatives that follow directly from derivatives of basic functions (power, exponential, logarithmic, and trigonometric). C.I.AI.A.1
2. Use substitution of variables to calculate antiderivatives (including changing limits for definite integrals). C.I.AI.A.2
3. Find specific antiderivatives using initial conditions. C.I.AI.A.3

B. Apply integrals to solve problems. C.I.AI.B

4. Use a definite integral to find the area of a region. C.I.AI.B.4
5. Use a definite integral to find the volume of a solid formed by rotating a region around a given axis. C.I.AI.B.5
6. Use integrals to solve a variety of problems (e.g., distance traveled by a particle along a line, exponential growth/decay). C.I.AI.B.6