Functions

Interpreting Functions Standards

Understand the concept of a function, and use function notation.

1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.1

Complexity a

a Determine if a relation is a function. F.IF.1.A

Complexity b

b Complete an input-output table when given the function rule and values. F.IF.1.B

Complexity c

c Identify the input or output of a function given in table form. F.IF.1.C

Learning Progression

- [Between b and c: F.IF.1.LP.A
- Interact with real-world situations that can be represented by functions. F.IF.1.LP.B
- Interact with relations that are not functions (e.g., Talk about pets, making Dog, Cat,and Horse as the input, ask the question "Which student has which pet?" and establish the correspondence, to decide if the relation is or is not a function.) F.IF.1.LP.C
- Establish the ordered pairs between the corresponding elements of the input and output.] F.IF.1.LP.D
- Show real world relations in table form. F.IF.1.LP.E
- Know the meaning of the words input and output. F.IF.1.LP.F
- Relate input and output to a function machine. F.IF.1.LP.G
- Understand that a table is made up of columns and rows. F.IF.1.LP.H
- Know that input is in the left column and the output is in right column. F.IF.1.LP.I
- Columns are up and down. F.IF.1.LP.J
- Rows are left and right. F.IF.1.LP.K
- Engagement Statements (demonstration of engaged in the topic) F.IF.1.LP.L
- Interact with constructing tables. F.IF.1.LP.M
- Interact with real-world situations. F.IF.1.LP.N
- Interact with a function machine. F.IF.1.LP.O
- Interact with technology. F.IF.1.LP.P

2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.2

Complexity a

a Given a linear equation using function notation, complete a table of values. F.IF.2.A

Complexity b

b Represent an equation in y= form with f(x). F.IF.2.B

Complexity c

c Understand that f(x)=y. F.IF.2.C

Learning Progression

- Experience real world concepts that are represented by a symbol. (e.g., street signs, symbols on technology tools, etc.) F.IF.2.LP.A
- Recognize that a function can be written as abstract mathematical symbolic language F.IF.2.LP.B
- Levels F.IF.1a-c need to be mastered. F.IF.2.LP.C
- Engagement Statements (demonstration of engaged in the topic) F.IF.2.LP.D
- Interact with constructing tables. F.IF.2.LP.E
- Interact with real-world situations. F.IF.2.LP.F
- Interact with a function machine. F.IF.2.LP.G
- Interact with technology. F.IF.2.LP.H

3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$. F.IF.3

Complexity a

a Given a sequence, determine the functional rule. F.IF.3.A

Complexity b

b Predict the next three terms in an arithmetic or geometric sequence (e.g., 3,6,9 …). F.IF.3.B

Complexity c

c Identify the common ratio or common difference in a sequence. F.IF.3.C

Learning Progression

- Describe the “rule” for a pattern. F.IF.3.LP.A
- Identify the “rule” for a pattern. F.IF.3.LP.B
- Discover patterns in the real world F.IF.3.LP.C
- Build or draw patterns using technology or manipulatives. F.IF.3.LP.D
- Translate patterns to numbers F.IF.3.LP.E
- Replicate a given pattern. F.IF.3.LP.F
- Engagement Statements (demonstration of engaged in the topic) F.IF.3.LP.G

- Interact with patterns `F.IF.3.LP.H`

Interpret functions that arise in applications in terms of the context.

4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (A2, M3) a. Focus on linear and exponential functions. (M1) b. Focus on linear, quadratic, and exponential functions. (A1, M2) F.IF.4

Complexity a

a Given a function made up of several linear functions, determine where the function is increasing, decreasing, or flat F.IF.4.A

Complexity b

b Given a graph of a linear equation, identify the y and/or x intercept. F.IF.4.B

Complexity c

c Determine whether the linear function is increasing, decreasing, or flat. F.IF.4.C

Learning Progression

- Identify the x- and y- axis. F.IF.4.LP.A
- Read the graph from left to right. F.IF.4.LP.B
- Recognize that the x- and y- axes are number lines. F.IF.4.LP.C
- Recognize a line. F.IF.4.LP.D
- Recognize patterns of the line going up or down, or staying at the same level, reading from left to right. F.IF.4.LP.E
- Experience the creation of graphs using science probes F.IF.4.LP.F
- Demonstrate stories of a given graph. F.IF.4.LP.G
- Create a graph to a story using technology. F.IF.4.LP.H
- Know that flat means not going up or down. F.IF.4.LP.I
- Know that increasing is up and decreasing is down. F.IF.4.LP.J
- Recognize that in real-world increasing is adding quantities. For example, buying more objects increases the price. Connecting these situations with the vocabulary of “positive slope”. F.IF.4.LP.K
- Recognize that in real-world decreasing is removing quantities. For example, buying more objects decreases the amount of money you have. Connecting these situations with the vocabulary of “negative slope”. F.IF.4.LP.L
- Recognize that in real-world there are situations where there is no change in the dependent variable. For example, an adult height does not change over time. F.IF.4.LP.M
- Connecting these situations with the vocabulary of “zero slope”. F.IF.4.LP.N

- Engagement Statements (demonstration of engaged in the topic) F.IF.4.LP.O
- Interact with real-world examples of lines. F.IF.4.LP.P

5 Relate the domain of a function to its graph, and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. a. Focus on linear and exponential functions. (M1) b. Focus on linear, quadratic, and exponential functions. (A1, M2) c. Emphasize the selection of a type of function for a model based on behavior of data and context. (A2, M3) F.IF.5

Complexity a

a Given the graph represented by a linear function, determine the domain. F.IF.5.A

Complexity b

b Given a context of a linear equation, describe the domain. F.IF.5.B

Complexity c

c Given a table, state the input values. F.IF.5.C

Learning Progression

- Given a real world situation, e.g., When I buy 1 chocolate bar - I pay 2 dollars, when I buy 2 chocolate bars - I pay 4 dollars, match the values in the situation with the values in the table. F.IF.5.LP.A
- Count physical objects. F.IF.5.LP.B
- Relate the input and the output to a function machine. F.IF.5.LP.C
- Answer questions about a table. F.IF.5.LP.D
- Identify what the input is in a table. F.IF.5.LP.E
- Engagement Statements (demonstration of engaged in the topic) F.IF.5.LP.F
- Interact with real-world situations that can be represented by functions. F.IF.5.LP.G
- Interact with constructing tables. F.IF.5.LP.H
- Interact with relations that are functions. F.IF.5.LP.I
- Explore a chart or a table. F.IF.5.LP.J

6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (A2, M3) F.IF.6

Complexity a

a Identify the slope of a line when the equation is written in slope intercept form. F.IF.6.A

Complexity b

b Identify the slope of a line when given in graph form. F.IF.6.B

Complexity c

c Determine whether a slope is present on a given visual graph. F.IF.6.C

Learning Progression

Not on BP

Analyze functions using different representations.

7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. a. Graph linear functions and indicate intercepts. (A1, M1) b. Graph quadratic functions and indicate intercepts, maxima, and minima. (A1, M2) c. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (A2, M3) d. Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior. (A2, M3) e. Graph simple exponential functions, indicating intercepts and end behavior. (A1, M1) f. Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (A2, M3) F.IF.7

Complexity a

a1 Graph a linear function using a graph with a scale of 1. F.IF.7.A1

a2 Determine whether an ordered pair is a viable solution to a given linear function. F.IF.7.A2

Complexity b

b1 Determine the y intercept point for a linear graph. F.IF.7.B1

b2 Determine whether the line is increasing (going up), decreasing (going down), or flat. F.IF.7.B2

Complexity c

c1 Identify two point on a linear graph. F.IF.7.C1

c2 Classify graphs of functions as linear or nonlinear. F.IF.7.C2

Learning Progression

- Identify the x- and y- axis. F.IF.7.LP.A
- Recognize that the x- and y- axes are number lines. F.IF.7.LP.B
- Recognize a line vs. a curve. F.IF.7.LP.C
- Draw lines and curves. F.IF.7.LP.D
- Use technology to explore graphing. F.IF.7.LP.E
- Recognize where the lines intersect the axes F.IF.7.LP.F
- Engagement Statements (demonstration of engaged in the topic) F.IF.7.LP.G
- Interact with graphs. F.IF.7.LP.H
- Interact with technology. F.IF.7.LP.I

Analyze functions using different representations.

8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3) i. Focus on completing the square to quadratic functions with the leading coefficient of 1. (A1, M2) b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of changeG in functions such as y = (1.02)t , and y = (0.97)t and classify them as representing exponential growth or decay. (A2, M3) i. Focus on exponential functions evaluated at integer inputs. (A1, M2) F.IF.8

Complexity a

a Identify equivalent expressions (e.g. 2x + 2x = 4x, x2 * x2 =x4). F.IF.8.A

Complexity b

b Identify equivalent expressions (limit to three terms) (e.g. x + x + x = 3x, x*x*x=x3). F.IF.8.B

Complexity c

c Identify equivalent expressions (limit to two terms) (e.g. x + x = 2x). F.IF.8.C

Learning Progression

Not on BP

9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (A2, M3) a. Focus on linear and exponential functions. (M1) b. Focus on linear, quadratic, and exponential functions. (A1, M2) F.IF.9

Complexity a

a Compare a function given in table form to another function given in graphical form. For example, which one is increasing? F.IF.9.A

Complexity b

b Match a function given in table form to its graph. F.IF.9.B

Complexity c

c Match a function given as a verbal description to its graph. For example, which is an increasing function? F.IF.9.C

Learning Progression

- Demonstrate stories to match an increasing/decreasing graph. F.IF.9.LP.A
- Match an increasing/decreasing graph to a story. F.IF.9.LP.B
- Connect to F.IF.4 and F.IF.3 F.IF.9.LP.C
- Engagement Statements (demonstration of engaged in the topic) F.IF.9.LP.D

- Interact with graphs. F.IF.9.LP.E
- Interact with technology. F.IF.9.LP.F
- Experience and act out real world situation that can be graphed (e.g., distance/time graphs). F.IF.9.LP.G

Building Functions Standards

Build a function that models a relationship between two quantities.

1 Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from context. i. Focus on linear and exponential functions. (A1, M1) ii. Focus on situations that exhibit quadratic or exponential relationships. (A1, M2) b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (A2, M3) c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time. F.BF.1

Complexity a

a Create a linear function that represents a linear relationship between quantities in a given context. F.BF.1.A

Complexity b

b Given a linear function that describes a realworld situation and given the value of one variable, find and interpret the value of the other variable. F.BF.1.B

Complexity c

c Identify the meaning of each number and/ or variable in a linear function that describes a realworld situation. F.BF.1.C

Learning Progression

- Count physical objects. F.BF.1.LP.A
- Relate the input and the output to a function machine. F.BF.1.LP.B
- Answer questions about a table. F.BF.1.LP.C
- Identify what the input is in a table. F.BF.1.LP.D
- Interpret variables and numbers in the linear function that describes the real - world situation F.BF.1.LP.E
- Know that a variable can represent an unknown value. F.BF.1.LP.F
- Identify a number sentence. F.BF.1.LP.G
- Recognize that the signs and numbers are not variables. F.BF.1.LP.H
- Identify numbers. F.BF.1.LP.I
- Relate a picture or objects to a number sentence. F.BF.1.LP.J
- Identify a number sentence. F.BF.1.LP.K
- Engagement Statements (demonstration of engaged in the topic) F.BF.1.LP.L
- Interact with no more than 3 answer choices be able to select 1 from different positions. F.BF.1.LP.M
- Interact with a visual model. F.BF.1.LP.N

- Interact with real-world situations described by linear functions F.BF.1.LP.O
- Interact with a model of a real-world situation. F.BF.1.LP.P
- Interact with representations of the unknown. F.BF.1.LP.Q

2 Write arithmetic and geometric sequences both recursively and with an explicit formula. Use them to model situations and translate between the two forms. F.BF.2

Complexity a

a Identify the rule for a pattern. F.BF.2.A

Complexity b

b Identify the next term in a pattern. F.BF.2.B

Complexity c

c Determine if a given set represents a pattern. F.BF.2.C

Learning Progression

- Experience patterns and non-patterns. F.BF.2.LP.A
- Discover patterns in the real world. F.BF.2.LP.B
- Build patterns using manipulatives. F.BF.2.LP.C
- Draw the patterns. F.BF.2.LP.D
- Translate the patterns to numbers. F.BF.2.LP.E
- Recognize the informal “rule” for the pattern. F.BF.2.LP.F
- Engagement Statements (demonstration of engaged in the topic) F.BF.2.LP.G
- Engage with patterns using manipulatives F.BF.2.LP.H

Build new functions from existing functions.

3 Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3) a. Focus on transformations of graphs of quadratic functions, except for f(kx. (A1, M2) F.BF.3

Complexity a

a Identify a line reflected over the y-axis on a coordinate grid. F.BF.3.A

Complexity b

b Identify a line reflected over the x-axis on a coordinate grid. F.BF.3.B

Complexity c

c Identify a line on a coordinate grid. F.BF.3.C

Learning Progression

Not on BP

4 Find inverse functions. a. Informally determine the input of a function when the output is known. (A1, M1) F.BF.4

Complexity a

a Solve for x when y is given (e.g. y = x+3; what is the value of x when y is 5?). F.BF.4.A

Complexity b

b Identify the input and output of a function. F.BF.4.B

Complexity c

c Identify a function. F.BF.4.C

Learning Progression

Not on BP

Linear Quadratic and Exponential Models Standards

Construct and compare linear, quadratic, and exponential models, and solve problems.

1 Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.LE.1

Complexity a

a Identify a situation that represents a linear and/or exponential function. F.LE.1.A

Complexity b

b Identify the graph of a linear function and an exponential function. F.LE.1.NB

Complexity c

c Identify a graph of a linear function. F.LE.1.NC

Learning Progression

- Sort graphs based on their shape. F.LE.1.NLP.A
- Draw lines and curves F.LE.1.NLP.B
- Recognize a line vs. a curve F.LE.1.NLP.C
- Observe graphs of different functions with technology F.LE.1.NLP.D
- Engagement Statements (demonstration of engaged in the topic F.LE.1.NLP.E

f. Interact with technology F.LE.1.NLP.F

- Interact with no more than 3 answer choices be able to select 1 from different positions. F.LE.1.NLP.G
- Interact with a visual model. F.LE.1.NLP.H

2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two inputoutput pairs (including reading these from a table). F.LE.2

Complexity a

a After creating a sequence, make a graph that represents that sequence. F.LE.2.A

Complexity b

b Create a geometric sequence of at least 5 numbers with models. F.LE.2.B

Complexity c

c Create an arithmetic sequence with a model. F.LE.2.C

Learning Progression

- Translate patterns to numbers F.LE.2.LP.A

- Build patterns using manipulatives F.LE.2.LP.B
- Draw the patterns F.LE.2.LP.C
- Recognize the informal “rule” for the pattern F.LE.2.LP.D
- Experience patterns and non-patterns. F.LE.2.LP.E
- Discover patterns in the real world. F.LE.2.LP.F
- Engagement Statements (demonstration of engaged in the topic F.LE.2.LP.G
- Engage with patterns using manipulatives F.LE.2.LP.H
- Interact with models of patterns. F.LE.2.LP.I

3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. (A1, M2) F.LE.3

Complexity a

a Observe a situation that shows increasing and decreasing linear and exponential events. (e.g., doubling a penny every day gives you more money than receiving $100 a day for a month) F.LE.3.A

Complexity b

b Identify an exponential function. F.LE.3.B

Complexity c

c Identify if a linear function is increasing or decreasing. F.LE.3.C

Learning Progression

Not on BP

4 For exponential models, express as a logarithm the solution to abct=d where a, c, and d are numbers, and the base b is 2, 10, or e; evaluate the logarithm using technology. F.LE.4

Complexity a

a Identify equivalent expressions with exponents. F.LE.4.A

Complexity b

b Identify equivalent expressions with exponents (limit to power 3 expressions) (e.g. Which is the same as m x m x m?). F.LE.4.B

Complexity c

c Identify equivalent expressions with exponents (limit to power 2 expressions) (e.g. Which is the same as m x m?). F.LE.4.C

Learning Progression

Not on BP

Interpret expressions for functions in terms of the situation they model.

5 Interpret the parameters in a linear or exponential function in terms of a context. F.LE.5

Complexity a

a Given a context, interpret the parameters of an exponential function (e.g., during x weeks, the existing number of fish in the pond has been doubled). This situation is modeled by the exponential function y = 100(2^x), where 100 is the initial number of fish in the pond, 2 is a growth factor, (2^x) is the number by which the initial number of fish, 100, is multiplied for every increase in x, and y is the total number of fish in the pond. F.LE.5.A

Complexity b

b Given a context, interpret the parameters of a linear function (e.g., Marsha has $10 already saved and saves an additional $5 a week for x number of weeks). This situation is modeled by a linear function f(x) = 5x +10, where 10 is the initial amount that has been saved, 5 is the weekly saving, 5x is the amount of money saved during x weeks, and f(x) is a total amount of money saved including the initial amount. F.LE.5.B

Complexity c

c Identify the constant in a linear or exponential equation. OR When given the graph of a function, identify the y intercept. F.LE.5.C

Learning Progression

Not on BP

Trigonometric Functions Standards

Extend the domain of trigonometric functions using the unit circle.

1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F.TF.1

Complexity a

a Identify the measure of a central angle on a circle when the measure of the arc is given. F.TF.1.A

Complexity b

b Identify the measure of an angle. F.TF.1.B

Complexity c

c Identify an angle. F.TF.1.C

Learning Progression

Not on BP

2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. F.TF.2

Complexity a

a Identify the measure of a central angle on a circle when the measure of the arc is given. F.TF.2.A

Complexity b

b Identify the measure of an angle. F.TF.2.B

Complexity c

c Identify an angle. F.TF.2.C

Learning Progression

Not on BP

Model periodic phenomena with trigonometric functions.

5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. F.TF.5

Complexity a

a Identify the measure of a central angle on a circle when the measure of the arc is given. F.TF.5.A

Complexity b

b Identify the measure of an angle. F.TF.5.B

Complexity c

c Identify an angle. F.TF.5.C

Learning Progression

Not on BP

Prove and apply trigonometric identities.

8 Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1, and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. F.TF.8

Complexity a

a Find the hypotenuse when the length of the sides is given. F.TF.5.LP.A

Complexity b

b Identify the parts of a right triangle. F.TF.5.LP.B

Complexity c

c Identify a right triangle. F.TF.5.LP.C

Learning Progression

Not on BP