North Dakota Mathematics Standards

# Grades 11, 12

Adopted 2023

## Eleventh and Twelfth Grades

### Math Attributes

Problem-Solving

P. Learners can analyze, execute, critique, and adapt approaches and solutions when problem-solving in novel situations. 11-12.MA.P

Connections

C. Learners can create connections within and across concepts, using supporting evidence to interpret how they originate, extend, and relate to other learning, ideas, and life experiences. 11-12.MA.C

Reasoning and Proof

R. Learners can reason logically, citing evidence to critique and explain what they see, think, and conclude through exploration, generalization, and validation. 11-12.MA.R

**Number and Operations**

2. Perform operations on complex radical expressions and simplify radicals to write equivalent expressions. 11-12.NO.2
3. Demonstrate that the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational. 11-12.NO.3
4. Use units to understand problems and to guide the solution of multi-step problems (e.g., unit analysis). Choose and interpret units consistently in formulas. Choose and interpret the scale and the units in graphs and data displays. 11-12.NO.4
5. Choose a level of accuracy or precision appropriate to limitations on measurement when reporting quantities. 11-12.NO.5
6. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real. Understand the hierarchal relationships among subsets of the complex number system. 11-12.NO.6
7. Use the definition $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. 11-12.NO.7
8. Use conjugates to find quotients of complex numbers. 11-12.NO.8
9. Apply the Fundamental Theorem of Algebra to determine the number of zeros for polynomial functions. 11-12.NO.9
10. Represent complex numbers on the complex plane in rectangular, trigonometric, and polar forms. Find the modulus (absolute value) of a complex number. Explain why the rectangular, trigonometric, and polar forms of a given complex number represent the same number. 11-12.NO.10
11. Represent addition, subtraction, multiplication, conjugation, powers, and roots of complex numbers geometrically on the complex and/or polar plane; use properties of this representation for computation. 11-12.NO.11
12. Extend polynomial identities to the complex numbers. 11-12.NO.12
13. Apply the Fundamental Theorem of Algebra to find all roots of a polynomial equation and determine the nature (i.e., integer, rational, irrational, real, complex) of the roots. 11-12.NO.13
14. Recognize vector quantities as having both magnitude and direction, writing them in polar form. 11-12.NO.14
15. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. 11-12.NO.15
16. Solve problems involving magnitude and direction that can be represented by vectors. 11-12.NO.16
17. Add and subtract vectors. Represent vector subtraction graphically by connecting the tips in the appropriate order and using the components to perform vector subtraction. 11-12.NO.17
    a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Know that the magnitude of a sum of two vectors is typically not the sum of the

magnitudes. 11-12.NO.17.A

b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. 11-12.NO.17.B

c. Understand that vector subtraction v-w is defined as v+(-w), where -w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. 11-12.NO.17.C

18. Multiply a vector by a scalar. 11-12.NO.18

a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction. Use the components to perform scalar multiplication (e.g., as c(v<sub>x</sub>, v<sub>y</sub>) = (cv<sub>x</sub> , cv<sub>y</sub>)). 11-12.NO.18.A

b. Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. 11-12.NO.18.B

c. Compute the direction of cv knowing that when |c|v ≠ 0, the direction of cv is either along v (for c > 0) or against v (for c < 0). 11-12.NO.18.C

19. Represent data in a matrix. Perform operations (i.e., addition, subtraction, multiplication) on matrices of appropriate dimensions to solve problems and in context. Know that matrix multiplication is not commutative. 11-12.NO.19

**Algebraic Reasoning**

1. Rearrange multi-variable formulas to highlight a quantity of interest. 11-12.AR.1
2. Use the structure of an expression (to extend to polynomial and rational expressions) to identify ways to rewrite it. 11-12.AR.2
3. Interpret expressions that represent a quantity in context. 11-12.AR.3
    a. Interpret parts of an expression, such as terms, factors, and coefficients. 11-12.AR.3.A
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. 11-12.AR.3.B
4. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. 11-12.AR.4
    a. Factor a quadratic expression to reveal the zeros of the function it defines. 11-12.AR.4.A
    b. Use the properties of exponents to transform exponential expressions. 11-12.AR.4.B
    c. Complete the square in a quadratic expression to produce an equivalent expression. 11-12.AR.4.C
5. Add, subtract, multiply, and divide rational expressions. Understand that rational expressions form a system analogous to rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression. 11-12.AR.5
6. Rewrite simple rational expressions in different forms. Write a(x)/b(x) in the form of q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or technology for the more complicated examples. 11-12.AR.6
7. Create equations and inequalities and use them to solve problems. Include equations arising from linear and quadratic equations and simple rational and exponential equations. 11-12.AR.7
8. Create equations in two or more variables to represent relationships between quantities. Graph equations on coordinate axes with proper labels and scales. 11-12.AR.8
9. Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context. 11-12.AR.9
10. Derive the quadratic formula from the form $0 = ax^2 + bx + c$. 11-12.AR.10
11. Solve quadratic equations with real coefficients that have solutions of the form a + bi and a - bi. 11-12.AR.11
12. Solve simple rational and radical equations in one variable and identify extraneous solutions. 11-12.AR.12
13. Add, subtract, and multiply polynomials beyond quadratics. Understand that polynomials form a system comparable to integers, namely, they are closed under

the operations of addition, subtraction, and multiplication. 11-12.AR.13

14. Identify zeros of polynomial equations when suitable factorizations are available. Use the zeros to construct a rough graph of the function defined by the polynomial. 11-12.AR.14
15. Apply the Factor and Remainder Theorems to determine efficiently whether a linear expression is a factor of a polynomial equation. Apply the Remainder Theorem in context. 11-12.AR.15
16. Using graphs, technology, tables, or successive approximations, show that the solution(s) to the equation f(x) = g(x) are the x-value(s) that result in the y-values of f(x) and g(x) being the same. 11-12.AR.16
17. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. 11-12.AR.17
18. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). 11-12.AR.18
19. Solve a system of equations in three or more variables with matrices (using technology). 11-12.AR.19
20. Apply the Binomial Theorem for the expansion of (ax + by)<sup>n</sup> in powers of x and y for a positive integer n and integers a and b. 11-12.AR.20

**Functions**

1. Write a function that describes a relationship between two quantities. 11-12.AR.F.1
    a. Combine standard function types using arithmetic operations. 11-12.AR.F.1.A
    b. Compose functions. 11-12.AR.F.1.B
2. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. 11-12.AR.F.2
3. Write a function defined by an expression in different but equivalent forms to reveal and explain the different properties of the function. 11-12.AR.F.3
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, minimum/maximum, and symmetry of the graph, and interpret these in terms of context. 11-12.AR.F.3.A
    b. Use the properties of exponents to interpret expressions for exponential functions. 11-12.AR.F.3.B
4. Identify the effect of transformations on the graph of a function by replacing f(x) with af(x), f(bx), f(x - h), and f(x) + k, for specific values of a, h, and k (both positive and negative). Find the value of a, b, h, and k given the graph of the function. Recognize even and odd functions from their graphs and equations. 11-12.AR.F.4
5. Find inverse functions. 11-12.AR.F.5
    a. Verify by composition that one function is the inverse of another. 11-12.AR.F.5.A
    b. Recognize that the graph of a function and its inverse are reflection images over the line y = x. 11-12.AR.F.5.B
    c. Produce an invertible function from a non-invertible function by restricting the domain. 11-12.AR.F.5.C
6. Apply the inverse relationship between exponents and logarithms to solve problems. 11-12.AR.F.6
7. Compare key features of two functions each represented in a different way (algebraically, graphically, numerically, in tables, or by verbal descriptions). 11-12.AR.F.7
8. Use tables, graphs, verbal descriptions, and equations to interpret and sketch the key features of a function modeling the relationship between two quantities. 11-12.AR.F.8
9. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. 11-12.AR.F.9
10. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. 11-12.AR.F.10
    a. Graph square root, cube root, piecewise-defined, step, and absolute value functions. 11-12.AR.F.10.A
    b. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. 11-12.AR.F.10.B

c. Graph exponential and logarithmic functions, showing intercepts and end behavior. 11-12.AR.F.10.C

d. Graph f(x) = sin x and f(x) = cos x as representations of periodic phenomena. 11-12.AR.F.10.D

11. Analyze and graph functions expressed symbolically (by hand in simple cases and using technology for more complicated cases), identifying key features of the graph. 11-12.AR.F.11

    a. Graph rational functions, identifying domain, range, asymptote(s), removable and non-removable discontinuities, intercepts, behavior at the asymptote(s), and end behavior. 11-12.AR.F.11.A

    b. Graph trigonometric functions, showing period, midline, phase shift, and amplitude. 11-12.AR.F.11.B

12. Compare the end behavior of linear, quadratic, and exponential functions using graphs and/or tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing as a linear or quadratic function. 11-12.AR.F.12

13. Determine whether a linear, quadratic, polynomial, exponential, logarithmic, or trigonometric model fits a situation. Determine an appropriate mathematical model in context (with or without technology). 11-12.AR.F.13

14. Write arithmetic and geometric sequences both recursively and with an explicit formula and convert between the two forms. Use sequences to model situations. 11-12.AR.F.14

15. Use properties of logarithms to express the solution to ab<sup>ct</sup> = d where a, c, and d are real numbers and b is a positive real number. Evaluate the logarithm using technology when appropriate. 11-12.AR.F.15

16. Extend right triangle trigonometry and apply knowledge of the unit circle to determine values of sine, cosine, and tangent for multiples of $\pi/3$, $\pi/4$, and $\pi/6$. 11-12.AR.F.16

17. Use the Pythagorean Identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to find sin (θ), cos (θ), or tan (θ) given sin (θ), cos (θ), or tan (θ) and the quadrant of the angle. 11-12.AR.F.17

18. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. 11-12.AR.F.18

19. Use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. 11-12.AR.F.19

20. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. 11-12.AR.F.20

21. Create a trigonometric function to model periodic phenomena. 11-12.AR.F.21

22. Restrict the domain of a trigonometric function to construct its inverse. 11-12.AR.F.22

23. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions and interpret them in context. 11-12.AR.F.23

24. Know and apply the addition and subtraction formulas for sine, cosine, and tangent to solve problems. 11-12.AR.F.24

**Geometry and Measurement**

1. Write the equation of a conic section given its special features. Convert between the standard form and general form equations of conic sections. 11-12.GM.1
2. Identify key features of a conic section given its equation. Apply properties of conic sections in context. 11-12.GM.2
3. Determine and apply appropriate formulas to solve right and non-right triangle problems in context. 11-12.GM.3
4. Derive the formula A = ½ ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. 11-12.GM.4

**Data, Probability, and Statistics**

1. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). 11-12.DPS.1
2. Use the mean and standard deviation of a data set to fit it to a normal distribution and estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. 11-12.DPS.2
3. Evaluate reports based on data. 11-12.DPS.3
    a. Identify and explain misleading use of data, recognize when claims based on data confuse correlation and causation. 11-12.DPS.3.A
    b. Recognize and describe how graphs and data can be distorted to support different points of view. 11-12.DPS.3.B
4. Represent data on a scatter plot for two quantitative variables and describe how the variables are related. 11-12.DPS.4
    a. Fit a function to the data (with or without technology) and interpret the special features (e.g., meaning of a and b in the exponential function y = ab<sup>x</sup>) of the function in context. 11-12.DPS.4.A
    b. Use functions fitted to data to solve problems in the context of the data. 11-12.DPS.4.B
5. Informally assess the fit of a function by plotting and analyzing residuals. 11-12.DPS.5
6. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. 11-12.DPS.6
7. Understand the process of making inferences about population parameters based on a random sample from that population. 11-12.DPS.7
8. Decide if a specified model is consistent with results from a given data-generating process (e.g., using simulation). 11-12.DPS.8
9. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. 11-12.DPS.9
10. Determine when the order in counting matters and use permutations and combinations to compute probabilities of events accordingly. Determine probability situations as conditional, "or" (union), or "and" (intersection), and determine the probability of an event. 11-12.DPS.10
11. Use permutations and combinations to compute probabilities of compound events and solve problems. 11-12.DPS.11
12. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space. Graph the corresponding probability distribution using the same graphical displays as for data distributions. 11-12.DPS.12
13. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. 11-12.DPS.13

14. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. Evaluate and compare strategies on the basis of expected values. 11-12.DPS.14

    a. Find the expected payoff for a game of chance. 11-12.DPS.14.A

15. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities are calculated; find the expected value. 11-12.DPS.15

16. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. 11-12.DPS.16

17. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). 11-12.DPS.17

18. Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). 11-12.DPS.18