Maryland Mathematics

Precalculus A

Number and Quantity N

DOMAIN THE COMPLEX NUMBER SYSTEM N.CN

Cluster Use complex numbers in polynomial identities and equations. C

8 Extend polynomial identities to the complex numbers. For example, rewrite + 2 x 4 as ( ) ( ) x ix i + − 2 2 . N.CN.C.8

. Clarifications/Examples: Apply the Complex Conjugate Theorem when:

. solving polynomial equations with degrees greater than or equal to two.

. analyzing the graph of a polynomial.

. building a polynomial given one complex root.

. Function families to which this standard applies:

. Polynomial Functions

. Rational Functions

9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. N.CN.C.9

. Clarifications/Examples:

. Apply knowledge of Fundamental Theorem of Algebra to solving polynomial equations of degree 3 and higher.

. Students should be able to identify existence of complex roots.

. Make connections between the nature of the roots of an equation and the behavior of the graph of the function.

. Function families to which this standard applies:

. Polynomial Functions

. Rational Functions

Algebra A

DOMAIN **SEEING STRUCTURE IN EXPRESSIONS** A.SSE

Cluster Interpret the structure of expressions. A

2 Use the structure of an expression to identify ways to rewrite it. A.SSE.A.2

. Clarifications/Examples:

. Use factors of polynomials to simplify/analyze rational expressions and the graphs of the related functions.

. Factor polynomial expressions, of degree three and higher, completely, over the complex number system (e.g. Factor $xx4 - 3xx2 - 28$ to $(xx2 + 4)(xx2 - 7)$ and then to .

. Rewrite trigonometric expressions based on algebraic structures .

. Discuss conjugates and how their structure can help when simplifying expressions, verifying identities and solving equations.

. Factor expressions to include using the sum and difference of cubes (e.g. Factor $xx6 - 27yy3$).

. Recognize and factor an expression in quadratic form (e.g. $ee2xx - 9eexx + 14$, $2\cos2\,xx - 3\cos xx + 1$, $xx6 - 9xx3 + 8$).

. Function families to which this standard applies:

. Logarithmic/Exponential Functions

. Polynomial Functions

. Radical Power Functions

. Rational Functions

. Trigonometric Functions

Cluster Write expressions in equivalent forms to solve problems. B

3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.SSE.B.3

. Clarifications/Examples: Produce an equivalent form of a rational expression to reveal information about the behavior of the graph of the related function.

. number and type of discontinuities

. zeros

. asymptotes and holes

. Clarifications/Examples: Rewrite complex fractions as rational expressions

. Function families to which this standard applies:

. Logarithmic/Exponential Functions

- Polynomial Functions
- Radical Power Functions
- Rational Functions
- Trigonometric Functions

3.c Use the properties of exponents to transform expressions for exponential functions. For example, the expression 11.1111^{t} can be rewritten as to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. A.SSE.B.3.C

- Clarifications/Examples:
  - Model in context.

  Function families to which this standard applies:
  - Logarithmic/Exponential Functions

DOMAIN ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS A.APR

Cluster Understand the relationship between zeroes and factors of polynomials. B

2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. A.APR.B.2

- Clarifications/Examples:
  - Factor polynomials to simplify rational expressions.
  - Use the Remainder Theorem to determine zeros (roots) of a polynomial.
- Function families to which this standard applies:
  - Polynomial Functions
  - Rational Functions

3.a Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. A.APR.B.3.A

- Clarifications/Examples:
  - Identify real zeros of polynomials.
  - Understand the relationship between the degree of a polynomial and the number and nature of the zeros of the polynomial.
- Function families to which this standard applies:
  - Polynomial Functions

Cluster Use polynomial identities to solve problems.

5 Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. A.APR.C.5

- Clarifications/Examples:
  - Limit to $n \leq 5$, and binomials with variables coefficient of one, or constants less than four.
- Function families to which this standard applies:
  - Polynomial Functions

Cluster Rewrite rational expressions. D

6 Rewrite simple rational expressions in different forms. A.APR.D.6

- Clarifications/Examples: Rational expressions have no restrictions on degree of the numerator or denominator.

- Clarifications/Examples: Understand the connection between rewriting rational expressions and using long division when factoring polynomials.
  - Apply the Rational Root Theorem.
- Function families to which this standard applies:
  - Polynomial Functions
  - Rational Functions

7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions. A.APR.D.7

- Clarifications/Examples:
  - Emphasize operations on rational expressions.
- Function families to which this standard applies:
  - Rational Functions

DOMAIN CREATING EQUATIONS A.CED

Cluster Create equations that describe numbers or relationships. A

1.a Create equations and inequalities in one variable and use them to solve problems. A.CED.A.1.A

- Clarifications/Examples:
  - Create equations and inequalities in one variable involving all algebraic and transcendental functions and piecewise defined functions that combine different types of functions.
  - Use equations and inequalities that arise from any type of function to solve problems.
- Function families to which this standard applies:
  - Logarithmic/Exponential Functions
  - Polynomial Functions
  - Radical Power Functions
  - Rational Functions
  - Trigonometric Functions

1b Create polynomial equations given roots. A.CED.A.1B

- Clarifications/Examples:
  - Use the Factor Theorem.
  - Use the Conjugate Root Theorem as it applies to irrationals.
- Function families to which this standard applies:
  - Polynomial Functions

3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. A.CED.A.3

- Clarifications/Examples:

  Understand why constraints exist for certain equations and inequalities and for systems of equations and inequalities.
  - Represent constraints of equations that contain composite expressions .
- Function families to which this standard applies:
  - Logarithmic/Exponential Functions
  - Polynomial Functions
  - Radical Power Functions
  - Rational Functions
  - Trigonometric Functions

DOMAIN REASONING WITH EQUATIONS AND INEQUALITIES A.REI

Cluster Solve equations and inequalities on one variable. B

B Solve equations and inequalities in one variable. A.REI.B

Clarifications/Examples:

- Include equations and inequalities that contain combinations of various algebraic and transcendental functions.
- Include equations that contain composite expressions.
- Solve equations in one variable algebraically, numerically and/or graphically.
- Justify solution methods.

Function families to which this standard applies:

- Logarithmic/Exponential Functions
- Polynomial Functions
- Radical Power Functions
- Rational Functions
- Trigonometric Functions

Cluster Solve systems of equations. C

7a Solve systems of equations comprised of various combinations of all algebraic and transcendental functions in two variables. A.REI.C.7A

Clarifications/Examples:

- Explore algebraic, numeric and graphical methods for solving systems of equations.
- Explore systems comprised of functions from two different function families (e.g. Solve the system $yy = xx2$ and $yy = 2 \cos xx$).

Function families to which this standard applies:

- Logarithmic/Exponential Functions
- Polynomial Functions
- Radical Power Functions
- Rational Functions
- Trigonometric Functions

Functions F

DOMAIN **INTERPRETING FUNCTIONS** F.IF

Cluster Understand the concept of a function and use function notation. A

2a Extend evaluating functions to include operations with composite functions, e.g. $ff(xx + 2) - ff(xx)$. F.IF.A.2A

Clarifications/Examples:

Build prerequisite skills needed to simplify difference quotient (e.g. Given $ff(xx) = xx2 + 3xx + 1$ find $ff(xx + h)$).

Function families to which this standard applies:

Logarithmic/Exponential Functions

Polynomial Functions

Radical Power Functions

Rational Functions

Trigonometric Functions

Cluster Interpret functions that arise in application in terms of context. B

4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities; sketch graphs showing key features given a verbal description of the relationship. F.IF.B.4

Clarifications/Examples:

Interpret the key features of the graph of any function in terms of context.

Explore rate of change over various size intervals.

Estimate points of inflection.

Describe the concavity on intervals.

Explain how to recognize asymptotic behavior given various representations of a function (algebraic, numeric and graphic).

Function families to which this standard applies:

Logarithmic/Exponential Functions

Polynomial Functions

Radical Power Functions

Rational Functions

Trigonometric Functions

5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.IF.B.5

Clarifications/Examples:

Discuss the domain of all function types including composite and inverse functions.

Function families to which this standard applies:

- Logarithmic/Exponential Functions
- Polynomial Functions
- Radical Power Functions
- Rational Functions
- Trigonometric Functions

Cluster Analyze functions using different representations. C

7f Graph all functions including piecewise-defined functions, step functions and absolute value functions F.IF.C.7F

Clarifications/Examples:

- Sketch polynomials of higher degree from factored form, using x- and y-intercepts, end behavior and degree.
- Describe end behavior using appropriate notation, i.e. given an equation, as $xx \to \pm\infty, ff(xx) \to \pm\infty$.
- Use information about end behavior of a polynomial to sketch and identify the possible degree of a polynomial.
- Graph inverse trigonometric functions and identify the related principal values.

Function families to which this standard applies:

- Logarithmic/Exponential Functions
- Polynomial Functions
- Radical Power Functions
- Rational Functions
- Trigonometric Functions

7g Determine the end behavior of the graph of a polynomial function using the degree and leading coefficient. F.IF.C.7G

Clarifications/Examples:

- Refer to the wording of the standard.

Function families to which this standard applies:

- Polynomial Functions

8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.C.8

Clarifications/Examples:

- Understand why a particular form of an expression would reveal properties such as zeros, extrema, intercepts, etc.
- Rewrite rational functions to reveal discontinuities.

Function families to which this standard applies:

- Logarithmic/Exponential Functions
- Polynomial Functions
- Radical Power Functions
- Rational Functions
- Trigonometric Functions

9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables or by a verbal description). F.IF.C.9

Clarifications/Examples:

- Extend student understanding by using more complex situations.

Function families to which this standard applies:

- Logarithmic/Exponential Functions
- Polynomial Functions
- Radical Power Functions
- Rational Functions
- Trigonometric Functions

DOMAIN BUILDING FUNCTIONS F.BF

Cluster Build a function that models a relationship between two quantities. A

1 Write a function that describes a relationship between two quantities, including more complex functions. F.BF.A.1

Clarifications/Examples:

Include piecewise defined functions comprised of different types of functions.

Include composite functions (e.g. $ff(xx) = \sin(2xx)$).

Function families to which this standard applies:

Logarithmic/Exponential Functions

Polynomial Functions

Radical Power Functions

Rational Functions

Trigonometric Functions

1c Compose functions. For example, if $TT(yy)$ is the temperature in the atmosphere as a function of height, and $h(tt)$ is the height of a weather balloon as a function of time, then is the temperature at the location of the weather balloon as a function of time. F.BF.A.1C

Clarifications/Examples:

Identify the domain and range of a composite function.

Recognize when a function is the composition of two simpler functions (e.g. $ff(xx) = ee\sin xx$).

Function families to which this standard applies:

Logarithmic/Exponential Functions

Polynomial Functions

Radical Power Functions

Rational Functions

Trigonometric Functions

Cluster Build new functions from existing functions. B

3 Identify the effect on the graph of replacing $ff(xx)$ by $ff(xx) + kk$, $kkkk(xx)$, $ff(kkkk)$, and $ff(xx + kk)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. F.BF.B.3

Clarifications/Examples:

Identify any function as an even, an odd function or neither given a graphic or algebraic representation.

Identify transformations from parent functions.

Function families to which this standard applies:

Logarithmic/Exponential Functions

Polynomial Functions

Trigonometric Functions

Radical Power Functions

Rational Functions

4 Find inverse functions. F.BF.B.4

Clarifications/Examples:

Find inverses of polynomials of the form $ff(xx) = aa(xx - h)nn + kk$.

Note: Read the blog, Inverse Functions: We're Teaching it all Wrong, before teaching inverses.

Function families to which this standard applies:

Logarithmic/Exponential Functions

Polynomial Functions

Radical Power Functions

Rational Functions

Trigonometric Functions

4b Verify by composition that one function is the inverse of another. F.BF.B.4B

Clarifications/Examples:

Refer to the wording of the standard.

Function families to which this standard applies:

Polynomial Functions

Radical Power Functions

Rational Functions

4c Read values of an inverse function from a graph or a table, given that the function has an inverse. F.BF.B.4C

Clarifications/Examples:

Given the numeric or graphic representation of an invertible function, produce the graphic and numeric representation of the inverse.

Interpret the reflection of a function over the line $yy = xx$ as a representation of the inverse of a function.

Function families to which this standard applies:

Logarithmic/Exponential Functions

Polynomial Functions

Radical Power Functions

Rational Functions

Trigonometric Functions

4d Produce an invertible function from a non-invertible function by restricting the domain. F.BF.B.4D

Clarifications/Examples:

Refer to the wording of the standard.

Function families to which this standard applies:

Polynomial Functions

Radical Power Functions

Trigonometric Functions

4e Build inverse trigonometric functions. F.BF.B.4E

Clarifications/Examples:

Build from unit circle.

Function families to which this standard applies:

Trigonometric Functions

5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. F.BF.B.5

Clarifications/Examples:

Solve exponential equations using logarithms.

Discuss relationships between domain, range, and asymptotes.

Function families to which this standard applies:

Logarithmic/Exponential Functions

DOMAIN **LINEAR, QUADRATIC, AND EXPONENTIAL MODELS** F.LE

Cluster Construct and compare linear, quadratic, and exponential models and solve problems. A

4a Use properties of logarithms, including both common and natural logarithms, to rewrite and solve exponential models. F.LE.A.4A

Clarifications/Examples:

Write logarithmic functions as inverses of exponential functions, including both common and natural logarithms.

Function families to which this standard applies:

Logarithmic/Exponential Functions

DOMAIN TRIGONOMETRIC FUNCTIONS T.TF

Cluster Extend the domain of trigonometric functions using the unit circle. A

2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. F.TF.A.2

Clarifications/Examples:

Define the six trigonometric functions in terms of coordinates from the unit circle.

Understand the relationship between right triangle trigonometric ratios and trigonometric functions.

Evaluate trigonometric functions using reference angles.

Function families to which this standard applies:

Trigonometric Functions

3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi\pi$ 3 , $\pi\pi$ 4 , $aaaaaa$ $\pi\pi$ 6 and use the unit circle to express the values of sine, cosine, and tangent for xx, $\pi\pi$ + xx, $aaaaaa$ $2\pi\pi$ − xx in terms of their values for x, where x is any real number. F.TF.A.3

Clarifications/Examples:

Refer to the wording of the standard.

Function families to which this standard applies:

Trigonometric Functions

4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. F.TF.A.4

Clarifications/Examples:

Write the domain of all six trigonometric functions.

Write the domain of trigonometric functions under transformations (e.g. yy = tan xx versus yy = tan(2xx)).

Function families to which this standard applies:

Trigonometric Functions

Cluster Model periodic phenomena with trigonometric functions. B

5 Choose trigonometric functions to model real world phenomena. F.TF.B.5

Clarifications/Examples:

Graph all six trigonometric functions.

Function families to which this standard applies:

Trigonometric Functions

6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. F.TF.B.6

Clarifications/Examples:

Refer to the wording of the standard.

Function families to which this standard applies:

Trigonometric Functions

7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in context. F.TF.B.7

Clarifications/Examples:

Refer to the wording of the standard.

Function families to which this standard applies:

Trigonometric Functions

Cluster Prove and apply trigonometric identities. C

9.a Use trigonometric identities to rewrite expressions and as a tool when solving trigonometric equations. F.TF.C.9.A

Clarifications/Examples:

Use the double angle and half angle identities for trigonometric functions to simplify, verify, and solve expressions and equations involving sine, cosine, and tangent.

Emphasize double angle identities are used most frequently in Calculus.

Prove trigonometric identities, including Pythagorean identities and even and odd identities, using a variety of strategies. Verify identities graphically.

Emphasize Pythagorean identities.

Function families to which this standard applies:

Trigonometric Functions

Geometry G

DOMAIN **SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY** G.SRT

Cluster Apply trigonometry to general triangles. D

10 Prove the Laws of Sines and Cosines and use them to solve problems. G.SRT.D.10

Clarifications/Examples:

Refer to the wording of the standard.

Function families to which this standard applies:

Trigonometric Functions

11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying, resultant forces). G.SRT.D.11

Clarifications/Examples:

Refer to the wording of the standard.

Function families to which this standard applies:

Trigonometric Functions

Statistics S

DOMAIN **INTERPRETING CATEGORICAL AND QUANTITATIVE DATA** S.ID

Cluster Summarize, represent, and interpret data on two categorical and quantitative variables. B

6d Fit a function to data represented by a scatterplot; use functions fitted to data to solve problems in the context of the data. S.ID.B.6D

Clarifications/Examples:

Determine the best function to represent data on two-quantitative variables by analyzing the context of the data; the behavior of the scatterplot and the fit of the function to the scatterplot.

Function families to which this standard applies:

Logarithmic/Exponential Functions

Polynomial Functions

Radical Power Functions

Trigonometric Functions

Functions F

DOMAIN **LINEAR, QUADRATIC, AND EXPONENTIAL MODELS** F.LE

Cluster Construct and compare linear, quadratic, and exponential models and solve problems. A

1.d Distinguish between situations that can be modeled with exponential functions and logistic functions. F.LE.A.1.D

Clarifications/Examples:

Refer to the wording of the standard.

Cluster Interpret expressions for functions in terms of the situation they model. B

5a Interpret the parameters in a logistic function in terms of a context. F.LE.B.5A

Clarifications/Examples:

Interpret the parameters A, B, and C in expressions of the form $yy = CC\ 1{+}AAee{-}BBBB$, in terms of a context.

6 Build and interpret logistic functions to model real-world problems. F.LE.B.6

Clarifications/Examples:

Refer to the wording of the standard.

6a Sketch and analyze the graphs of logistic functions. F.LE.B.6A

Clarifications/Examples:

Refer to the wording of the standard.

6b Compare and contrast the exponential, logarithmic, and logistic models. F.LE.B.6B

Clarifications/Examples:

Refer to the wording of the standard.

6c Apply understanding of logarithmic and logistic functions to solve real-world problems. F.LE.B.6C

Clarifications/Examples:

Refer to the wording of the standard.