Kentucky Mathematics

Grades 9, 10, 11, 12

Adopted 2019

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.** MP.1
2. **Reason abstractly and quantitatively.** MP.2
3. **Construct viable arguments and critique the reasoning of others.** MP.3
4. **Model with mathematics.** MP.4
5. **Use appropriate tools strategically.** MP.5
6. **Attend to precision.** MP.6
7. **Look for and make use of structure.** MP.7
8. **Look for and express regularity in repeated reasoning.** MP.8

Number and Quantity-The Real Number System

Extend the properties of exponents to rational exponents.

1. Extend the properties of integer exponents to rational exponents, allowing for the expression of radicals in terms of rational exponents. KY.HS.N.1
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. KY.HS.N.2

Use properties of rational and irrational numbers.

3. (+) Justify why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. KY.HS.N.3

Number and Quantity-Quantities

Reason quantitatively and use units to solve problems.

4. Use units in context as a way to understand problems and to guide the solution of multi-step problems; KY.HS.N.4
    a. Choose and interpret units consistently in formulas; KY.HS.N.4.A
    b. Choose and interpret the scale and the origin in graphs and data displays. KY.HS.N.4.B
5. Define appropriate units in context for the purpose of descriptive modeling. KY.HS.N.5
6. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. KY.HS.N.6

Number and Quantity-The Complex Number System

Perform arithmetic operations with complex numbers.

7. Understanding properties of complex numbers. KY.HS.N.7
    a. Know there is a complex number i such that $i^2 = -1$ and every complex number has the form a + bi with a and b real. KY.HS.N.7.A
    b. Use the relation $i^2 = -1$ and the commutative, associative and distributive properties to add, subtract and multiply complex numbers. KY.HS.N.7.B
    c. (+) Find the conjugate of a complex number and use it to find the quotient of complex numbers. KY.HS.N.7.C

Represent complex numbers and their operations on the complex plane.

8. (+)Understanding representations of complex numbers using the complex plane. KY.HS.N.8
    a. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number. KY.HS.N.8.A
    b. Represent addition, subtraction, multiplication, modulus and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. KY.HS.N.8.B
    c. Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints. KY.HS.N.8.C

Use complex numbers in polynomial identities and equations.

9. Solve quadratic equations with real coefficients that have complex solutions. KY.HS.N.9
10. (+) Extend polynomial identities to the complex numbers. KY.HS.N.10
11. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. KY.HS.N.11

Number and Quantity-Vector and Matrix Quantities

Represent and model with vector quantities.

12. (+) Understand and apply properties of vectors. KY.HS.N.12

    a. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes. KY.HS.N.12.A

    b. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. KY.HS.N.12.B

    c. Solve problems involving velocity and other quantities that can be represented by vectors. KY.HS.N.12.C

Perform operations on vectors.

13. (+) Perform operations with vectors (addition, subtraction and multiplication by a scalar). KY.HS.N.13

    a. Add vectors end-to-end, component-wise and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. KY.HS.N.13.A

    b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. KY.HS.N.13.B

    c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order and perform vector subtraction component-wise. KY.HS.N.13.C

    d. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. KY.HS.N.13.D

    e. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$). KY.HS.N.13.E

Perform operations on matrices and use matrices in applications.

14. Use matrices to represent and manipulate data. KY.HS.N.14
15. Perform operations with matrices. KY.HS.N.15
    a. Add, subtract and multiply matrices of appropriate dimensions. KY.HS.N.15.A
    b. Multiply matrices by scalars to produce new matrices. KY.HS.N.15.B
16. (+) Understand properties of square and identity matrices. KY.HS.N.16
    a. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. KY.HS.N.16.A
    b. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. KY.HS.N.16.B
    c. Work with 2 × 2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area. KY.HS.N.16.C
17. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. KY.HS.N.17

Algebra

Seeing Structure in Expressions

Interpret the structure of expressions.

1. interpret expressions that represent a quantity in terms of its context. KY.HS.A.1
   a. Interpret parts of an expression, such as terms, factors and coefficients. KY.HS.A.1.A
   b. Interpret complicated expressions, given a context, by viewing one or more of their parts as a single entity. KY.HS.A.1.B
2. Use the structure of an expression to identify ways to rewrite it and consistently look for opportunities to rewrite expressions in equivalent forms. KY.HS.A.2

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. KY.HS.A.3
   a. Write the standard form of a given polynomial and identify the terms, coefficients, degree, leading coefficient and constant term. KY.HS.A.3.A
   b. Factor a quadratic expression to reveal the zeros of the function it defines. KY.HS.A.3.B
   c. Use the properties of exponents to rewrite exponential expressions. KY.HS.A.3.C
   d. (+) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. KY.HS.A.3.D
4. (+) Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. KY.HS.A.4

Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.

5. Add, subtract and multiply polynomials. KY.HS.A.5

Understand the relationship between zeros and factors of polynomials.

6. (+) Know and apply the Remainder Theorem. KY.HS.A.6
7. Identify roots of polynomials when suitable factorizations are available. Know these roots become the zeros (x-intercepts) for the corresponding polynomial function. KY.HS.A.7

Use polynomial identities to solve problems.

8. (+) Prove polynomial identities and use them to describe numerical relationships. KY.HS.A.8
9. (+) Know and apply the Binomial Theorem for the expansion of (x + y) ⁿ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. KY.HS.A.9

Rewrite rational expressions.

10. (+) Rewrite simple rational expressions in different forms. KY.HS.A.10
11. (+) Add, subtract, multiply and divide rational algebraic expressions. KY.HS.A.11

Creating Equations

Create equations that describe numbers or relationships.

12. Create equations and inequalities in one variable and use them to solve problems. KY.HS.A.12
13. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. KY.HS.A.13
14. Create a system of equations or inequalities to represent constraints within a modeling context. Interpret the solution(s) to the corresponding system as viable or nonviable options within the context. KY.HS.A.14
15. Rearrange formulas to solve a literal equation, highlighting a quantity of interest, using the same reasoning as in solving equations. KY.HS.A.15

Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

16. Understand each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. KY.HS.A.16

17. Solve and justify equations in one variable. Justify the solutions and give examples showing how extraneous solutions may arise. KY.HS.A.17

    a. Solve rational equations written as proportions in one variable. KY.HS.A.17.A

    b. Solve radical equations in one variable. KY.HS.A.17.B

Solve equations and inequalities in one variable.

18. Solve linear equations and inequalities in one variable, including literal equations with coefficients represented by letters. KY.HS.A.18

19. Solve quadratic equations in one variable. KY.HS.A.19

    a. Solve quadratic equations by taking square roots, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. KY.HS.A.19.A

    b. (+) Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. KY.HS.A.19.B

    c. (+) Solve quadratic equations by completing the square. KY.HS.A.19.C

Solve systems of equations.

20. Solve systems of linear equations in two variables. KY.HS.A.20

    a. Understand a system of two equations in two variables has the same solution as a new system formed by replacing one of the original equations with an equivalent equation. KY.HS.A.20.A

    b. Solve systems of linear equations with graphs, substitution and elimination, focusing on pairs of linear equations in two variables. KY.HS.A.20.B

21. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. KY.HS.A.21

22. (+) Use matrices to solve a system of equations. KY.HS.A.22

    a. Represent a system of linear equations as a single matrix equation in a vector variable. KY.HS.A.22.A

    b. Find the inverse of a matrix if it exists. KY.HS.A.22.B

    c. Use matrices to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater). KY.HS.A.22.C

Represent and solve equations and inequalities graphically.

23. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. KY.HS.A.23

24. Justify that the solutions of the equations f(x) = g(x) are the x-coordinates of the points where the graphs of y = f(x) and y = g(x) intersect. Find the approximate solutions graphically, using technology or tables. KY.HS.A.24

25. Graph linear inequalities in two variables. KY.HS.A.25

    a. Graph the solutions to a linear inequality as a half-plane (excluding the boundary in the case of a strict inequality). KY.HS.A.25.A

    b. Graph the solution set to a system of linear inequalities as the intersection of the corresponding half-planes. KY.HS.A.25.B

Functions

Interpreting Functions

Understand the concept of a function and use function notation.

1. Understand properties and key features of functions and the different ways functions can be represented. KY.HS.F.1
   a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. KY.HS.F.1.A
   b. Using appropriate function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. KY.HS.F.1.B
   c. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. KY.HS.F.1.C
   d. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. KY.HS.F.1.D
   e. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). KY.HS.F.1.E
2. Recognize that arithmetic and geometric sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. KY.HS.F.2

Interpret functions that arise in applications in terms of the context.

3. Understand average rate of change of a function over an interval. KY.HS.F.3
   a. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. KY.HS.F.3.A
   b. Estimate the rate of change from a graph. KY.HS.F.3.B

Analyze functions using different representations.

4. Graph functions expressed symbolically and show key features of the graph, with and without using technology (computer, graphing calculator). KY.HS.F.4
   a. Graph linear and quadratic functions and show intercepts, maxima and minima. KY.HS.F.4.A
   b. Graph square root, cube root and absolute value functions. KY.HS.F.4.B
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. KY.HS.F.4.C
   d. Graph exponential and logarithmic functions, showing intercepts and end behavior. KY.HS.F.4.D
   e. (+) Graph trigonometric functions, showing period, midline and amplitude. KY.HS.F.4.E
   f. (+) Graph piecewise functions, including step functions. KY.HS.F.4.F

g. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior. KY.HS.F.4.G

5. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. KY.HS.F.5

   a. Identify zeros, extreme values and symmetry of the graph within the context of a quadratic function. KY.HS.F.5.A

   b. Use the properties of exponents to interpret expressions for exponential functions and classify the exponential function as representing growth or decay. KY.HS.F.5.B

Building Functions

Build a function that models a relationship between two quantities.

6. Write a function that describes a relationship between two quantities. KY.HS.F.6
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context. KY.HS.F.6.A
   b. Combine standard function types using arithmetic operations. KY.HS.F.6.B
   c. (+) Compose functions. KY.HS.F.6.C
7. Use arithmetic and geometric sequences to model situations and scenarios. KY.HS.F.7
   a. Use formulas (explicit and recursive) to generate terms for arithmetic and geometric sequences. KY.HS.F.7.A
   b. Write formulas to model arithmetic and geometric sequences and apply those formulas in realistic situations. KY.HS.F.7.B
   c. (+) Translate between recursive and explicit formulas. KY.HS.F.7.C

Build new functions from existing functions.

8. Understand the effects of transformations on the graph of a function. KY.HS.F.8
   a. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx) and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. KY.HS.F.8.A
   b. Experiment with cases and illustrate an explanation of the effects on the graph using technology. KY.HS.F.8.B
9. Find inverse functions. KY.HS.F.9
   a. Given the equation of an invertible function, find the inverse. KY.HS.F.9.A
   b. (+) Verify by composition that one function is the inverse of another. KY.HS.F.9.B
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. KY.HS.F.9.C
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain. KY.HS.F.9.D
10. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents with the use of technology. KY.HS.F.10

Linear, Quadratic and Exponential Functions

Construct and compare linear, quadratic and exponential models and solve problems.

11. Distinguish between situations that can be modeled with linear functions and with exponential functions. KY.HS.F.11

    a. Recognize and justify that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. KY.HS.F.11.A

    b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. KY.HS.F.11.B

    c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. KY.HS.F.11.C

12. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). KY.HS.F.12

13. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. KY.HS.F.13

Interpret expressions for functions in terms of the situation they model.

14. Interpret the parameters in a linear or exponential function in terms of a context. KY.HS.F.14

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

15. (+) Understand the relationship of radian measure of an angle to its arc length. KY.HS.F.15
16. (+) Understand and use the unit circle. KY.HS.F.16
    a. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. KY.HS.F.16.A
    b. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$ and use the unit circle to express the values of sine, cosine and tangent for $\pi - x$, $\pi + x$ and $2\pi - x$ in terms of their values for x, where x is any real number. KY.HS.F.16.B
    c. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. KY.HS.F.16.C

Model periodic phenomena with trigonometric functions.

17. (+) Choose trigonometric functions to model periodic phenomena with specified period, midline and amplitude. KY.HS.F.17
18. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. KY.HS.F.18
19. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context. KY.HS.F.19

Prove and apply trigonometric identities.

20. (+) Proving identities and formulas within the context of trigonometry. KY.HS.F.20
    a. Prove the Pythagorean identity and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. KY.HS.F.20.A
    b. Prove the addition and subtraction formulas for sine, cosine and tangent and use them to solve problems. KY.HS.F.20.B

Geometry

Congruence

Experiment with transformations in the plane.

1. Know and apply precise definitions of the language of Geometry: KY.HS.G.1
   a. Understand properties of line segments, angles and circle. KY.HS.G.1.A
   b. Understand properties of and differences between perpendicular and parallel lines. KY.HS.G.1.B
2. Representing transformations in the plane. KY.HS.G.2
   a. Describe transformations as functions that take points in the plane as inputs and give other points as outputs KY.HS.G.2.A
   b. Compare transformations that preserve distance and angle measures to those that do not. KY.HS.G.2.B
   c. Given a rectangle, parallelogram, trapezoid, or regular polygon, formally describe the rotations and reflections that carry it onto itself, using properties of these figures. KY.HS.G.2.C
3. (+) Develop formal definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments. KY.HS.G.3
4. Understand the effects of transformations of geometric figures. KY.HS.G.4
   a. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure. KY.HS.G.4.A
   b. Specify a sequence of transformations that will carry a given figure onto another. KY.HS.G.4.B
   c. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure. Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. KY.HS.G.4.C

Understand congruence in terms of rigid motions.

5. Know and apply the concepts of triangle congruence: KY.HS.G.5
   a. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. KY.HS.G.5.A
   b. Explain how the criteria for triangle congruence (ASA, SAS and SSS) follow from the definition of congruence in terms of rigid motions. KY.HS.G.5.B

Prove geometric theorems.

6. Apply theorems for lines, angles, triangles, parallelograms. KY.HS.G.6
7. Prove theorems about geometric figures. KY.HS.G.7
   a. Construct formal proofs to justify theorems for lines, angles and triangles. KY.HS.G.7.A
   b. (+) Construct formal proofs to justify theorems for parallelograms. KY.HS.G.7.B

Make geometric constructions.

8. Create and apply geometric constructions. KY.HS.G.8
    a. Make formal geometric constructions with a variety of tools and methods. KY.HS.G.8.A
    b. Apply basic construction procedures to construct more complex figures. KY.HS.G.8.B

Similarity, Right Triangles and Trigonometry

Understand similarity in terms of similarity transformations.

9. Understand properties of dilations. KY.HS.G.9
    a. Verify the properties that result from that dilations given by a center and a scale factor. KY.HS.G.9.A
    b. Verify that a dilation produces an image that is similar to the pre-image. KY.HS.G.9.B
10. Apply the properties of similarity transformations to establish the AA criterion for two triangles to be similar. KY.HS.G.10

Prove theorems involving similarity.

11. Understand theorems about triangles. KY.HS.G.11
    a. Apply theorems about triangles. KY.HS.G.11.A
    b. (+) Prove theorems about triangles. KY.HS.G.11.B
    c. Use similarity criteria for triangles to solve problems and to prove relationships in geometric figures. KY.HS.G.11.C

Define trigonometric ratios and solve problems involving right triangles.

12. Understand properties of right triangles. KY.HS.G.12
    a. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles (sine, cosine and tangent). KY.HS.G.12.A
    b. Explain and use the relationship between the sine and cosine of complementary angles. KY.HS.G.12.B
    c. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. KY.HS.G.12.C

Apply trigonometry to general triangles.

13. (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. KY.HS.G.13
14. (+) Understand and apply the Law of Sines and the Law of Cosines. KY.HS.G.14
    a. Use the Law of Sines and Cosines to find unknown measurements in right and non-right triangles. KY.HS.G.14.A
    b. Prove the Laws of Sines and Cosines and use them to solve problems. KY.HS.G.14.B

Circles

Understand and apply theorems about circles.

15. Verify using dilations that all circles are similar. KY.HS.G.15
16. Identify and describe relationships among angles and segments within the context of circles involving: KY.HS.G.16
    a. Recognize differences between and properties of inscribed, central and circumscribed angles. KY.HS.G.16.A
    b. Understand relationships between inscribed angles and the diameter of a circle. KY.HS.G.16.B
    c. Understand the relationship between the radius of a circle and the line drawn through the point of tangency on that radius. KY.HS.G.16.C
17. (+) Apply basic construction procedures within the context of a circle. KY.HS.G.17
    a. Construct the inscribed and circumscribed circles of a triangle. KY.HS.G.17.A
    b. Construct a tangent line from a point outside a given circle to the circle. KY.HS.G.17.B

Find arc lengths and areas of sectors of circles.

18. (+) Understand the relationship between an intercepted arc length within a circle and the radius of the circle. KY.HS.G.18
    a. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius. Derive the formula for the area of a sector. KY.HS.G.18.A
    b. Define the radian measure of the angle as the measure of a central angle that intercepts an arc equal in length to the radius of the circle. KY.HS.G.18.B

Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section.

19. Understand the relationship between the algebraic form and the geometric representation of a circle. KY.HS.G.19
    a. Write the equation of a circle of given center and radius using the Pythagorean Theorem. KY.HS.G.19.A
    b. (+) Derive and write the equation of a circle of given center and radius using the Pythagorean Theorem. KY.HS.G.19.B
    c. (+) Complete the square to find the center and radius of a circle given by an equation. KY.HS.G.19.C
20. (+) Derive the equations of conic sections. KY.HS.G.20
    a. Derive the equation of a parabola given a focus and directrix. KY.HS.G.20.A
    b. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. KY.HS.G.20.B

Use coordinates to prove simple geometric theorems algebraically.

21. Use coordinates to justify and prove simple geometric theorems algebraically. KY.HS.G.21
22. Justify and apply the slope criteria for parallel and perpendicular lines and use them to solve geometric problems. KY.HS.G.22
23. Find measurements among points within the coordinate plane. KY.HS.G.23
    a. Use points from the coordinate plane to find the coordinates of a midpoint of a line segment and the distance between the endpoints of a line segment. KY.HS.G.23.A
    b. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. KY.HS.G.23.B
24. Use coordinates within the coordinate plane to calculate measurements of two dimensional figures. KY.HS.G.24
    a. Compute the perimeters of various polygons. KY.HS.G.24.A
    b. Compute the areas of triangles, rectangles and other quadrilaterals. KY.HS.G.24.B

Geometric Measurement and Dimensions

Explain volume formulas and use them to solve problems.

25. Analyze and determine the validity of arguments for the formulas for the various figures and shapes. KY.HS.G.25
    a. Finding the circumference and area of a circle. KY.HS.G.25.A
    b. Finding the volume of a sphere, prism, cylinder, pyramid and cone. KY.HS.G.25.B
26. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. KY.HS.G.26
27. Use volume formulas to solve problems for cylinders, pyramids, cones, spheres, prisms. KY.HS.G.27

Visualize relationships between two-dimensional and three-dimensional objects.

28. Identify the shapes of two-dimensional cross-sections of three-dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects. KY.HS.G.28

Modeling with Geometry

Apply geometric concepts in modeling situations.

29. Use geometric shapes, their measures and their properties to describe objects in real world settings. KY.HS.G.29
30. Apply concepts of density based on area and volume in modeling situations, using appropriate units of measurement. KY.HS.G.30
31. Apply geometric methods to solve design problems. KY.HS.G.31

Statistics and Probability

Interpreting Categorical and Quantitative Data

Summarize, represent and interpret data on a single count or measurement variable.

1. Represent the distribution of data with plots on the real number line (stem plots, dot plots, histograms and box plots). KY.HS.SP.1
2. Use statistics appropriate to the shape of the numerical data distribution to compare center (median, mean) and spread (interquartile range when comparing medians and standard deviation when comparing means) of different data distributions. KY.HS.SP.2
3. Interpret differences in shape, center and spread in the context of the distributions of the numerical data, accounting for the presence and possible effects of extreme data points (outliers). KY.HS.SP.3
4. (+) When appropriate, fit a normal distribution to a numerical data set for given mean and standard deviation and then estimate population percentages using the Empirical Rule and recognize that there are data sets for which such a procedure is not appropriate. KY.HS.SP.4

Summarize, represent and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two or more categories in frequency tables. Calculate and interpret joint, marginal and conditional relative frequencies (probabilities) in the context of the data, recognizing possible associations and trends in the data. KY.HS.SP.5
6. Represent data on two quantitative variables on a scatter plot and describe how the explanatory and response variables are related. KY.HS.SP.6
    a. Calculate an appropriate mathematical model, or use a given mathematical model, for data to solve problems in context. KY.HS.SP.6.A
    b. Informally assess the fit of a model (through calculating correlation for linear data, plotting, calculating and/or analyzing residuals). KY.HS.SP.6.B

Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. KY.HS.SP.7
8. Understand the role and purpose of correlation in linear regression. KY.HS.SP.8
    a. Use technology to compute correlation coefficient of a linear fit. KY.HS.SP.8.A
    b. Interpret the meaning of the correlation within the context of the data. KY.HS.SP.8.B
    c. Describe the limitations of correlation when establishing causation. KY.HS.SP.8.C

Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.

9. Understand statistics as a process for making inferences and justifying conclusions about population parameters based on a random sample from that population. KY.HS.SP.9
10. Decide if a specified model is consistent with the results from a simulation. KY.HS.SP.10

Make inferences and justify conclusions from sample surveys, experiments and observational studies.

11. Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each. KY.HS.SP.11
12. Use data from a sample survey to estimate a population mean or proportion and explain how bias may be involved in the process. KY.HS.SP.12
13. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between estimates or statistics are significant. KY.HS.SP.13

Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data.

14. Describe events as subsets of a sample space. Use characteristics (or categories) of the outcomes, such as,
as unions, "A or B," that are mutually exclusive events and
as unions, "A or B," that are non-mutually exclusive events and
as intersections, "A and B," and
as complements of other events, "not A."
to calculate basic probabilities. KY.HS.SP.14
15. Understand the concept of independence. KY.HS.SP.15
    a. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their individual probabilities, P(A) x P(B) KY.HS.SP.15.A
    b. (+) Determine whether two events are independent and provide a justification to support the decision. KY.HS.SP.15.B
    c. Recognize and explain the concept of independence in everyday language and everyday situations. KY.HS.SP.15.C
16. Understand the concept of conditional probability. KY.HS.SP.16
    a. Understand the conditional probability of A given B as P(A and B)/P(B). KY.HS.SP.16.A
    b. (+) Interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A and the conditional probability of B given A is the same as the probability of B. KY.HS.SP.16.B
    c. Recognize and explain the concept of conditional probability in everyday language and everyday situations. KY.HS.SP.16.C
    d. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model. KY.HS.SP.16.D
17. (+) Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide whether events are independent and to approximate conditional probabilities. KY.HS.SP.17

Use the rules of probability to compute probabilities of compound events.

18. (+) Apply the General Multiplication Rule, P(A and B) = P(A)P(B|A) = P(B)P(A|B), in a uniform probability model and interpret the answer in terms of the model. KY.HS.SP.18
19. Use permutations and combinations to compute probabilities. KY.HS.SP.19
    a. Distinguish between situations that can be modeled using counting techniques, including Fundamental Counting Principle, permutations and combinations. KY.HS.SP.19.A

b. Perform calculations using the appropriate counting technique, including simple probabilities. KY.HS.SP.19.B

c. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. KY.HS.SP.19.C

Using Probability to Make Decisions

Calculate expected values and use them to solve problems.

20. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same appropriate graphical displays as for data distributions. KY.HS.SP.20

21. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution and use the value in analyzing decisions. KY.HS.SP.21

22. (+) Develop a probability distribution for a random variable. KY.HS.SP.22

    a. Find an expected value based on a sample space in which theoretical probabilities can be calculated. KY.HS.SP.22.A

    b. Find an expected value based on a sample space in which empirical probabilities can be calculated. KY.HS.SP.22.B

Use probability to evaluate outcomes of decisions.

23. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. KY.HS.SP.23

    a. Find the expected payoff for a game of chance. KY.HS.SP.23.A

    b. Evaluate and compare strategies on the basis of expected values. KY.HS.SP.23.B

    c. Use calculated expected values to make fair decisions and formulate strategies. KY.HS.SP.23.C

Calculus

Limits

Understand the concept of the limit of a function.

1. (+) Understand limits. KY.HS.C.1
    a. Apply limits to a variety of functions, including piecewise functions. KY.HS.C.1.A
    b. (++) Prove that the limit of a function exists, based upon the definition of a limit. KY.HS.C.1.B
2. (+) Demonstrate an understanding of limits by estimating and finding the limit of a function at a point graphically, numerically and algebraically. KY.HS.C.2
3. (+) Apply properties and theorems of limits, including limits of indeterminate forms. KY.HS.C.3
4. (+) Communicate understanding of limits using precise mathematical symbols and language. KY.HS.C.4

Function Behavior

Describe the asymptotic and unbounded behavior of functions.

5. (+) Describe asymptotic behavior (analytically and graphically) in terms of infinite limits and limits at infinity. KY.HS.C.5
6. (+) Discuss the end behavior of functions; identify representative functions for each type of end behavior using precise mathematical symbols and language. KY.HS.C.6

Continuity

Develop an understanding of continuity as a property of functions.

7. (+) Understand and use the limit definition of continuity to determine whether a given function is continuous at a specific point. KY.HS.C.7
8. (+) Define and identify different types of discontinuity – removable (hole) or non-removable (jump, asymptote) – in terms of limits. KY.HS.C.8
9. (+) Understand and apply continuous function theorems. KY.HS.C.9
    a. Apply the Intermediate Value Theorem to continuous functions. KY.HS.C.9.A
    b. Apply the Extreme Value Theorem to continuous functions. KY.HS.C.9.B
10. (+) Communicate an understanding of continuity using precise mathematical symbols and language. KY.HS.C.10

Understanding the Derivative

Demonstrate an understanding of the derivative.

11. (+) Define derivatives. KY.HS.C.11
    a. Define the derivative of a function as the limit of the difference quotient. KY.HS.C.11.A
    b. Understand this limit of the difference quotient can be interpreted as an instantaneous rate of change or the slope of a tangent line. KY.HS.C.11.B
12. (+) Use average rate of change to estimate the derivative from a table of values or a graph. KY.HS.C.12
13. (+) Understand the derivative as a function. KY.HS.C.13
14. (+) Apply the definition of derivative to find derivative values and derivative functions. KY.HS.C.14
15. (+) Explain why differentiability implies continuity yet continuity does not imply differentiability. KY.HS.C.15
16. (+) Understand and apply the Mean Value Theorem, including numerical, graphical and algebraic representations. KY.HS.C.16
17. (+) Understand the relationship between the concavity of a function and the sign of the second derivative. KY.HS.C.17
18. (++) Understand Rolle's Theorem as a special case of the Mean Value Theorem. KY.HS.C.18

Applications of Derivatives

Apply differentiation techniques.

19. (+) Efficiently find derivatives of functions with and without technology. KY.HS.C.19
20. (+) Understand and use derivative rules for sums, differences, products and quotients of two functions and calculate the derivative of a composite function using the chain rule. KY.HS.C.20
21. (+) Use implicit differentiation to find a derivative in an equation of two variables. KY.HS.C.21
22. (+) Use implicit differentiation to find the derivative of the inverse of a function. KY.HS.C.22
23. (+) Understand the relationship between the increasing and decreasing behavior of a function and the sign of the first derivative of the function. KY.HS.C.23
24. (+) Use the first derivative to analyze curves and identify relative extrema. KY.HS.C.24
25. (+) Understand the relationship of concavity to the second derivative. KY.HS.C.25
26. (+) Use the second derivative to find points of inflection. KY.HS.C.26
27. (+) Use the second derivative to analytically locate intervals on which a function is concave up, concave down or neither. KY.HS.C.27
28. (+) Describe how graphical characteristics of a given function, the first derivative of that function and the second derivative of that function interrelate. KY.HS.C.28
29. (+) Use derivatives to express rate of change in a variety of contexts. KY.HS.C.29
30. (+) Use derivatives to solve a variety of problems including related rates, optimization, tangent line approximations and growth and decay models. KY.HS.C.30
31. (+) Use differentiation to solve problems involving velocity, speed and acceleration. KY.HS.C.31
32. (+) Understand and apply differential equations. KY.HS.C.32
    a. Verify solutions to differential equations and use them to model real-world problems with and without technology. KY.HS.C.32.A
    b. Solve separable differential equations and use them in modeling real-world problems with and without technology. KY.HS.C.32.B

Understanding Integration

Demonstrate understanding of a definite integral.

33. (+) Understand the definite integral of a function over an interval. Interpret a definite integral as a limit of Riemann Sums and as net accumulation of change. KY.HS.C.33
34. (+) Write a Riemann sum that represents the definition of a definite integral. KY.HS.C.34
35. (+) Calculate the values of Riemann Sums over equal subdivisions to approximate definite integrals of functions represented graphically and numerically (using tables). Use left-hand sums, right-hand sums, midpoint sums and trapezoidal sums. KY.HS.C.35
36. (+) Recognize differentiation and integration as inverse operations. KY.HS.C.36
37. (+) Understand how the Fundamental Theorem of Calculus connects differentiation and integration and use it to evaluate definite and indefinite integrals and to represent particular antiderivatives. KY.HS.C.37
38. (+) Perform analytical and graphical analysis of functions using the Fundamental Theorem of Calculus. KY.HS.C.38
39. (+) Understand and use the definite integral of a function over an interval and understand its use as a mathematical tool. KY.HS.C.39

Applications of Integration

Apply techniques of integration.

40. (+) Find antiderivatives of a variety of basic functions including power, exponential, logarithmic and trigonometric and apply basic properties of definite integrals. KY.HS.C.40
41. (+) Use substitution techniques and change of limits of integration to find antiderivatives. KY.HS.C.41
42. (+) Find particular antiderivatives given initial conditions. KY.HS.C.42

Define trigonometric ratios and solve problems involving right triangles.

43. (+) Model, solve and interpret applications of antiderivatives including finding area, velocity, acceleration and volume of a solid. KY.HS.C.43
44. (+) Apply integration to solve problems including particle motion and exponential growth and decay. KY.HS.C.44
45. (+) Describe the application of integration to a variety of problems using precise mathematical language and notation. KY.HS.C.45