Kansas Mathematics

Grades 9, 10

Adopted 2017

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.** MP.1
2. **Reason abstractly and quantitatively.** MP.2
3. **Construct viable arguments and critique the reasoning of others.** MP.3
4. **Model with mathematics** MP.4
5. **Use appropriate tools strategically** MP.5
6. **Attend to precision.** MP.6
7. **Look for and make use of structure.** MP.7
8. **Look for and express regularity in repeated reasoning.** MP.8

Number and Quantity

The Real Number System

Use properties of rational numbers and irrational numbers.

1. Know and apply the properties of integer exponents to generate equivalent numerical and algebraic expressions. N.RN.1

Quantities

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N.Q.1
2. Define appropriate quantities for the purpose of descriptive modeling. N.Q.2
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. N.Q.3

The Complex Number System

Perform arithmetic operations with complex numbers.

4. Use conjugates to find moduli and quotients of complex numbers. N.CN.4

Represent complex numbers and their operations on the complex plane.

5. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. N.CN.5
6. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. N.CN.6
7. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. N.CN.7

Use complex numbers in polynomial identities and equations.

9. Extend polynomial identities to the complex numbers. N.CN.9
10. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. N.CN.10

Vector and Matrix Quantities

Represent and model with vector quantities.

1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v). N.VM.1
2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. N.VM.2
3. Solve problems involving velocity and other quantities that can be represented by vectors. N.VM.3

Perform operations on vectors.

4. Add and subtract vectors. N.VM.4
    a. Add vectors end-to-end, component-wise, and by the parallelogram rule . Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. N.VM.4.A
    b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. N.VM.4.B
    c. Understand vector subtraction v - w as v + (-w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. N.VM.4.C
5. Multiply a vector by a scalar. N.VM.5
    a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, (e.g. as c(v_x, v_y) = (cv_x, cv_y).) N.VM.5.A
    b. Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when |c|v ≠ 0, the direction of cv is either along v (for c > 0) or against v (for c < 0). N.VM.5.B

Perform operations on matrices and use matrices in applications.

9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. N.VM.9
10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. N.VM.10
11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. N.VM.11

Algebra

Seeing Structure in Expressions

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context. A.SSE.1
   a. Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.1.A
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. A.SSE.1.B
2. Use the structure of an expression to identify ways to rewrite it. A.SSE.2

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.SS.3
   a. Factor a quadratic expression to reveal the zeros of the function it defines. A.SS.3.A

Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.

1. Add, subtract, and multiply polynomials. A.APR.1

Use polynomial identities to solve problems.

4. Generate polynomial identities from a pattern. A.APR.4
5. Know and apply the Binomial Theorem for the expansion of $(x + y)^2$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. The Binomial Theorem can be proven by mathematical induction or by a combinatorial argument. A.APR.5

   Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. A.APR.6
7. Add, subtract, multiply, and divide rational expressions. A.APR.7

Creating Equations

Create equations that describe numbers or relationships.

1. Apply and extend previous understanding to create equations and inequalities in one variable and use them to solve problems. A.CED.1
2. Apply and extend previous understanding to create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.2
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. A.CED.3
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. A.CED.4

Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. A.REI.1

Solve equations and inequalities in one variable.

2. Apply and extend previous understanding to solve equations, inequalities, and compound inequalities in one variable, including literal equations and inequalities. (A.REI.3) A.REI.2
3. Solve equations in one variable and give examples showing how extraneous solutions may arise. A.REI.3
    a. Solve rational, absolute value and square root equations. Limited to simple equations such as, $2\sqrt{x-3} + 8 = 16$, $x + 3/2x - 1 = 5$, $x \neq ½$. A.REI.3.A
    b. Solve exponential and logarithmic equations. A.REI.3.B
5. Solve quadratic equations and inequalities A.REI.5
    a. Solve quadratic equations by inspection (e.g. for $x^2 = 49$), taking square roots, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives no real solutions. A.REI.5.A

Solve systems of equations.

6. Analyze and solve pairs of simultaneous linear equations. A.REI.6
    a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. A.REI.6.A
    b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. A.REI.6.B
    c. Solve real-world and mathematical problems leading to two linear equations in two variables. A.REI.6.C
7. Represent a system of linear equations as a single matrix equation and solve (incorporating technology) for matrices of dimension 3 × 3 or greater. A.REI.7

Represent and solve equations and inequalities graphically.

8. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A.REI.8
9. Solve an equation $f(x) = g(x)$ by graphing $y = f(x)$ and $y = g(x)$ and finding the x-value of the intersection point. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. A.REI.9

10. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. A.REI.10

Functions

Interpreting Functions

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.1
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.2
3. Recognize patterns in order to write functions whose domain is a subset of the integers. Limited to linear and quadratic. F.IF.3

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of expressions, graphs and tables in terms of the quantities, and sketch graphs showing key features given a description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. F.IF.4
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.IF.5
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. F.IF.6

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.7
    a. Graph linear, quadratic and absolute value functions and show intercepts, maxima, minima and end behavior. F.IF.7.A
    d. Graph piecewise-defined functions, including step functions. F.IF.7.D
    f. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. F.IF.7.F
    g. Graph trigonometric functions, showing period, midline, and amplitude. F.IF.7.G
8. Write a function in different but equivalent forms to reveal and explain different properties of the function. F.IF.8

a. Use different forms of linear functions, such as slope-intercept, standard, and point-slope form to show rate of change and intercepts. F.IF.8.A

9. Compare properties of two functions using a variety of representations (algebraically, graphically, numerically in tables, or by verbal descriptions). F.IF.9

Building Functions

Build a function that models a relationship between two quantities.

1. Use functions to model real-world relationships. F.BF.1
   a. Combine multiple functions to model complex relationships. F.BF.1.A
2. Write arithmetic and geometric sequences and series both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F.BF.2

Build new functions from existing functions.

3. Transform parent functions (f(x)) by replacing f(x) with f(x)+k, kf(x), f(kx), and f(x+k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. F.BF.3
4. Find inverse functions. F.BF.4
   c. Verify by composition that one function is the inverse of another. F.BF.4.C
   d. Produce an invertible function from a non-invertible function by restricting the domain. F.BF.4.D

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F.TF.1
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. F.TF.2
3. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. F.TF.3
4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. F.TF.4

Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. F.TF.5
6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. F.TF.6
7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. F.TF.7

Prove and apply trigonometric identities.

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant. F.TF.8
9. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. F.TF.9

Geometry

Congruence

Experiment with transformations in the plane.

1. Verify experimentally (for example, using patty paper or geometry software) the properties of rotations, reflections, translations, and symmetry: G.CO.1
   a. Lines are taken to lines, and line segments to line segments of the same length. G.CO.1.A
   b. Angles are taken to angles of the same measure. G.CO.1.B
   c. Parallel lines are taken to parallel lines. G.CO.1.C
   d. Identify any line and/or rotational symmetry within a figure. G.CO.1.D
2. Recognize transformations as functions that take points in the plane as inputs and give other points as outputs and describe the effect of translations, rotations, and reflections on two-dimensional figures. G.CO.2

Understand congruence in terms of rigid motions.

3. Given two congruent figures, describe a sequence of rigid motions that exhibits the congruence (isometry) between them using coordinates and the non-coordinate plane. G.CO.3
4. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G.CO.4
5. Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G.CO.5
6. Demonstrate triangle congruence using rigid motion (ASA, SAS, and SSS). G.CO.6

Construct arguments about geometric theorems using rigid transformations and/or logic.

7. Construct arguments about lines and angles using theorems. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. G.CO.7
8. Construct arguments about the relationships within one triangle using theorems. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point; angle sum and exterior angle of triangles. G.CO.8
9. Construct arguments about the relationships between two triangles using theorems. Theorems include: SSS, SAS, ASA, AAS, and HL. G.CO.9
10. Construct arguments about parallelograms using theorems. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. G.CO.10

Make geometric constructions.

11. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. G.CO.11

12. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. G.CO.12

Similarity, Right Triangles, and Trigonometry

Understand similarity in terms of similarity transformations.

1. Use geometric constructions to verify the properties of dilations given by a center and a scale factor: G.SRT.1
   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. G.SRT.1.A
   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. G.SRT.1.B
2. Recognize transformations as functions that take points in the plane as inputs and give other points as outputs and describe the effect of dilations on two-dimensional figures. G.SRT.2
3. Given two similar figures, describe a sequence of transformations that exhibits the similarity between them using coordinates and the non-coordinate plane. G.SRT.3
4. Understand the meaning of similarity for two-dimensional figures as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. G.SRT.4

Construct arguments about theorems involving similarity.

5. Construct arguments about triangles using theorems. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity, and AA. G.SRT.5
6. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. G.SRT.6

Define trigonometric ratios and solve problems involving right triangles.

7. Show that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G.SRT.7
8. Explain and use the relationship between the sine and cosine of complementary angles. G.SRT.8
9. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. G.SRT.9

Apply trigonometry to general triangles

10. Derive the formula A = ½ ab sin C for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. G.SRT.10
11. Prove the Laws of Sines and Cosines and use them to solve problems. G.SRT.11
12. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g. surveying problems, resultant forces). G.SRT.12

Circles

Understand and apply theorems about circles.

1. Construct arguments that all circles are similar. G.C.1
2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. G.C.2
3. Construct arguments using properties of polygons inscribed and circumscribed about circles. G.C.3
4. Construct inscribed and circumscribed circles for triangles. G.C.4
5. Construct inscribed and circumscribed circles for polygons and tangent lines from a point outside a given circle to the circle. G.C.5

Find arc lengths and areas of sectors of circles.

6. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. G.C.6

Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section.

1. Write the equation of a circle given the center and radius or a graph of the circle; use the center and radius to graph the circle in the coordinate plane. G.GPE.1
2. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; graph the circle in the coordinate plane; G.GPE.2
3. Complete the square to find the center and radius of a circle given by an equation. G.GPE.3
4. Derive the equation of a parabola given a focus and directrix; graph the parabola in the coordinate plane. G.GPE.4
5. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant; graph the ellipse or hyperbola in the coordinate plane. G.GPE.5

Use coordinates to prove simple geometric theorems algebraically.

6. Use coordinates to prove simple geometric theorems algebraically, including the use of slope, distance, and midpoint formulas For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle. G.GPE.6
7. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g. find the equation of a line parallel or perpendicular to a given line that passes through a given point). G.GPE.7
8. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, including the use of the distance and midpoint formulas. G.GPE.8

Geometric Measurement and Dimension

Explain volume formulas and use them to solve problems.

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments and informal limit arguments. G.GMD.1
2. Give an informal argument using Cavalieri's principle for the formulas for the volume of a solid figure. G.GMD.2

Modeling with Geometry

Apply geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g. modeling a tree trunk or a human torso as a cylinder). G.MG.1
2. Apply concepts of density and displacement based on area and volume in modeling situations (e.g. persons per square mile, BTUs per cubic foot). G.MG.2
3. Apply geometric methods to solve design problems (e.g. designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). G.MG.3

Statistics & Probability

Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

1. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S.ID.1
2. Interpret differences in shape, center, and spread in the context of the data sets using dot plots, histograms, and box plots, accounting for possible effects of extreme data points (outliers). S.ID.2
3. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. S.ID.3

Summarize, represent, and interpret data on two categorical and quantitative variables.

4. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. S.ID.4
5. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. S.ID.5

a. Use a given linear function to solve problems in the context of data. S.ID.5.A

b. Fit a linear function to data and use it to solve problems in the context of the data. S.ID.5.B

c. Assess the fit of a function by plotting and analyzing residuals. S.ID.5.C

d. Fit quadratic and exponential functions to the data. Use functions fitted to data to solve problems in the context of the data. S.ID.5.D

Interpret linear models.

6. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. S.ID.6

Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.

1. Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population. S.IC.1
2. Decide if a specified model is consistent with results from a given data-generating process, e.g. using simulation. S.IC.2

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S.IC.3
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error, (e.g. through the use of simulation models for random sampling.) S.IC.4
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. S.IC.5
6. Evaluate reports based on data. S.IC.6

Conditional Probability and the Rules of Probability

Understand independent and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). S.CP.1
2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S.CP.2
3. Understand the conditional probability of A given B as P(A and B)/P(B) , and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. S.CP.3
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. S.CP.4
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. S.CP.5

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. S.CP.6
7. Apply the Addition Rule, P(A or B) = P(A)+P(B)-P(A and B), and interpret the answer in terms of the model. S.CP.7
8. Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B | A) = P(B)P(A | B), and interpret the answer in terms of the model. S.CP.8
9. Use permutations and combinations to compute probabilities of compound events and solve problems. S.CP.9

Using Probability to Make Decisions

Calculate expected values and use them to solve problems.

1. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. S.MD.1
2. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. S.MD.2
3. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. S.MD.3
4. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. S.MD.4

Use probability to evaluate outcomes of decisions.

5. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. S.MD.5
   a. Find the expected payoff for a game of chance. S.MD.5.A
   b. Evaluate and compare strategies on the basis of expected values. S.MD.5.B
6. Use probabilities to make fair decisions (e.g. drawing by lots, using a random number generator). S.MD.6
7. Analyze decisions and strategies using probability concepts (e.g. product testing, medical testing, pulling a hockey goalie at the end of a game). S.MD.7