Kansas Mathematics

Grade 12

Adopted 2017

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.** MP.1
2. **Reason abstractly and quantitatively.** MP.2
3. **Construct viable arguments and critique the reasoning of others.** MP.3
4. **Model with mathematics** MP.4
5. **Use appropriate tools strategically** MP.5
6. **Attend to precision.** MP.6
7. **Look for and make use of structure.** MP.7
8. **Look for and express regularity in repeated reasoning.** MP.8

Number and Quantity

Quantities

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N.Q.1
2. Define appropriate quantities for the purpose of descriptive modeling. N.Q.2
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. N.Q.3

The Complex Number System

Perform arithmetic operations with complex numbers.

4. Use conjugates to find moduli and quotients of complex numbers. N.CN.4

Represent complex numbers and their operations on the complex plane.

5. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. N.CN.5
6. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. N.CN.6
7. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. N.CN.7

Use complex numbers in polynomial identities and equations.

9. Extend polynomial identities to the complex numbers. N.CN.9
10. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. N.CN.10

Vector and Matrix Quantities

Represent and model with vector quantities.

1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v). N.VM.1
2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. N.VM.2
3. Solve problems involving velocity and other quantities that can be represented by vectors. N.VM.3

Perform operations on vectors.

4. Add and subtract vectors. N.VM.4
   a. Add vectors end-to-end, component-wise, and by the parallelogram rule . Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. N.VM.4.A
   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. N.VM.4.B
   c. Understand vector subtraction v - w as v + (-w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. N.VM.4.C
5. Multiply a vector by a scalar. N.VM.5
   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, (e.g. as c(v_x, v_y) = (cv_x, cv_y).) N.VM.5.A
   b. Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when |c|v ≠ 0, the direction of cv is either along v (for c > 0) or against v (for c < 0). N.VM.5.B

Perform operations on matrices and use matrices in applications.

9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. N.VM.9
10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. N.VM.10
11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. N.VM.11

Algebra

Seeing Structure in Expressions

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context. A.SSE.1

   a. Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.1.A

   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. A.SSE.1.B

2. Use the structure of an expression to identify ways to rewrite it. A.SSE.2

Arithmetic with Polynomials and Rational Expressions

Use polynomial identities to solve problems.

5. Know and apply the Binomial Theorem for the expansion of $(x + y)^2$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. The Binomial Theorem can be proven by mathematical induction or by a combinatorial argument. A.APR.5

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. A.APR.6

7. Add, subtract, multiply, and divide rational expressions. A.APR.7

Creating Equations

Create equations that describe numbers or relationships.

1. Apply and extend previous understanding to create equations and inequalities in one variable and use them to solve problems. A.CED.1

2. Apply and extend previous understanding to create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.2

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. A.CED.3

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. A.CED.4

Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. A.REI.1

Solve equations and inequalities in one variable.

2. Apply and extend previous understanding to solve equations, inequalities, and compound inequalities in one variable, including literal equations and inequalities. (A.REI.3) A.REI.2

Solve systems of equations.

7. Represent a system of linear equations as a single matrix equation and solve (incorporating technology) for matrices of dimension 3×3 or greater. A.REI.7

Represent and solve equations and inequalities graphically.

8. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A.REI.8

Functions

Interpreting Functions

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.1
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.2

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of expressions, graphs and tables in terms of the quantities, and sketch graphs showing key features given a description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. F.IF.4
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.IF.5

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.7
    - d. Graph piecewise-defined functions, including step functions. F.IF.7.D
    - f. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. F.IF.7.F
    - g. Graph trigonometric functions, showing period, midline, and amplitude. F.IF.7.G
8. Write a function in different but equivalent forms to reveal and explain different properties of the function. F.IF.8
9. Compare properties of two functions using a variety of representations (algebraically, graphically, numerically in tables, or by verbal descriptions). F.IF.9

Building Functions

Build a function that models a relationship between two quantities.

1. Use functions to model real-world relationships. F.BF.1
2. Write arithmetic and geometric sequences and series both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F.BF.2

Build new functions from existing functions.

4. Find inverse functions. F.BF.4
   c. Verify by composition that one function is the inverse of another. F.BF.4.C
   d. Produce an invertible function from a non-invertible function by restricting the domain. F.BF.4.D

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F.TF.1
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. F.TF.2
3. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. F.TF.3
4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. F.TF.4

Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. F.TF.5
6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. F.TF.6
7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. F.TF.7

Prove and apply trigonometric identities.

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant. F.TF.8
9. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. F.TF.9

Geometry

Congruence

Understand congruence in terms of rigid motions.

5. Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G.CO.5

6. Demonstrate triangle congruence using rigid motion (ASA, SAS, and SSS). G.CO.6

Make geometric constructions.

12. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. G.CO.12

Similarity, Right Triangles, and Trigonometry

Construct arguments about theorems involving similarity.

5. Construct arguments about triangles using theorems. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity, and AA. G.SRT.5

6. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. G.SRT.6

Apply trigonometry to general triangles

10. Derive the formula A = ½ ab sin C for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. G.SRT.10

11. Prove the Laws of Sines and Cosines and use them to solve problems. G.SRT.11

12. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g. surveying problems, resultant forces). G.SRT.12

Circles

Understand and apply theorems about circles.

4. Construct inscribed and circumscribed circles for triangles. G.C.4

5. Construct inscribed and circumscribed circles for polygons and tangent lines from a point outside a given circle to the circle. G.C.5

Find arc lengths and areas of sectors of circles.

6. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. G.C.6

Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section.

2. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; graph the circle in the coordinate plane; `G.GPE.2`
3. Complete the square to find the center and radius of a circle given by an equation. `G.GPE.3`
4. Derive the equation of a parabola given a focus and directrix; graph the parabola in the coordinate plane. `G.GPE.4`
5. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant; graph the ellipse or hyperbola in the coordinate plane. `G.GPE.5`

Statistics & Probability

Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

3. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. `S.ID.3`

Summarize, represent, and interpret data on two categorical and quantitative variables.

c. Assess the fit of a function by plotting and analyzing residuals. `S.ID.5.C`

d. Fit quadratic and exponential functions to the data. Use functions fitted to data to solve problems in the context of the data. `S.ID.5.D`

Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.

1. Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population. S.IC.1
2. Decide if a specified model is consistent with results from a given data-generating process, e.g. using simulation. S.IC.2

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S.IC.3
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error, (e.g. through the use of simulation models for random sampling.) S.IC.4
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. S.IC.5
6. Evaluate reports based on data. S.IC.6

Conditional Probability and the Rules of Probability

Understand independent and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). S.CP.1

2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S.CP.2

3. Understand the conditional probability of A given B as P(A and B)/P(B) , and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. S.CP.3

4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. S.CP.4

5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. S.CP.5

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. S.CP.6

7. Apply the Addition Rule, P(A or B) = P(A)+P(B)-P(A and B), and interpret the answer in terms of the model. S.CP.7

8. Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. S.CP.8

9. Use permutations and combinations to compute probabilities of compound events and solve problems. S.CP.9

Using Probability to Make Decisions

Calculate expected values and use them to solve problems.

1. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. S.MD.1
2. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. S.MD.2
3. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. S.MD.3
4. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. S.MD.4

Use probability to evaluate outcomes of decisions.

5. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. S.MD.5

a. Find the expected payoff for a game of chance. S.MD.5.A

b. Evaluate and compare strategies on the basis of expected values. S.MD.5.B

6. Use probabilities to make fair decisions (e.g. drawing by lots, using a random number generator). S.MD.6
7. Analyze decisions and strategies using probability concepts (e.g. product testing, medical testing, pulling a hockey goalie at the end of a game). S.MD.7