Idaho Mathematics

Grades 9, 10, 11, 12

Adopted 2022

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.** MP.1
2. **Reason abstractly and quantitatively.** MP.2
3. **Construct viable arguments and critique the reasoning of others.** MP.3
4. **Model with mathematics.** MP.4
5. **Use appropriate tools strategically.** MP.5
6. **Attend to precision.** MP.6
7. **Look for and make use of structure.** MP.7
8. **Look for and express regularity in repeated reasoning.** MP.8

The Real Number System

A. Extend the properties of exponents to rational exponents. 9-12.RNS.A

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. 9-12.RNS.A.1
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. 9-12.RNS.A.2

B. Use properties of rational and irrational numbers. 9-12.RNS.B

3. Explain why the sum or product of two rational numbers is rational; why the sum of a rational number and an irrational number is irrational; and why the product of a nonzero rational number and an irrational number is irrational. 9-12.RNS.B.3

Quantities

A. Reason quantitatively and use units to solve problems. 9-12.Q.A

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. 9-12.Q.A.1
2. Define appropriate quantities for the purpose of descriptive modeling. 9-12.Q.A.2
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. 9-12.Q.A.3

The Complex Number System

A. Perform arithmetic operations with complex numbers. 9-12.N.CN.A

1. Know there is a complex number i such that $i^2 = -1$, and show that every complex number has the form a + bi where a and b real. 9-12.N.CN.A.1
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. 9-12.N.CN.A.2
3. (+) Find the conjugate of a complex number; use conjugates to find absolute value and quotients of complex numbers. 9-12.N.CN.A.3

B. Represent complex numbers and their operations on the complex plane. 9-12.N.CN.B

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. 9-12.N.CN.B.4
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. 9-12.N.CN.B.5
6. (+) Calculate the distance between numbers in the complex plane as the absolute value of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. 9-12.N.CN.B.6

C. Use complex numbers in polynomial identities and equations. 9-12.N.CN.C

7. Solve quadratic equations with real coefficients that have complex solutions. 9-12.N.CN.C.7
8. (+) Extend polynomial identities to the complex numbers. 9-12.N.CN.C.8
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. 9-12.N.CN.C.9

Vector and Matrix Quantities

A. Represent and model with vector quantities. 9-12.VM.A

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v). 9-12.VM.A.1
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. 9-12.VM.A.2
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors. 9-12.VM.A.3

B. Perform operations on vectors. 9-12.VM.B

4. (+) Add and subtract vectors. 9-12.VM.B.4
   a. (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. 9-12.VM.B.4.A
   b. (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. 9-12.VM.B.4.B
   c. (+) Demonstrate understanding of vector subtraction v – w as v + (–w), where -- w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. 9-12.VM.B.4.C
5. (+) Multiply a vector by a scalar. 9-12.VM.B.5
   a. (+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy). 9-12.VM.B.5.A
   b. (+) Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv, knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$). 9-12.VM.B.5.B

C. Perform operations on matrices and use matrices in applications. 9-12.VM.C

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. 9-12.VM.C.6
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. 9-12.VM.C.7
8. (+) Add, subtract, and multiply matrices of appropriate dimensions. 9-12.VM.C.8
9. (+) Demonstrate understanding that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. 9-12.VM.C.9
10. (+) Demonstrate understanding that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. 9-12.VM.C.10
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. 9-12.VM.C.11
12. (+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. 9-12.VM.C.12

Algebra

Seeing Structure in Expressions

A. Interpret the structure of linear, quadratic, exponential, polynomial, and rational expressions. A.SSE.A

1. Interpret expressions that represent a quantity in terms of its context. A.SSE.A.1
    a. Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.A.1.A
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. A.SSE.A.1.B
2. Use the structure of an expression to identify ways to rewrite it. A.SSE.A.2

B. Write expressions in equivalent forms to solve problems. A.SSE.B

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.SSE.B.3
    a. Factor a quadratic expression to reveal the zeros of the function it defines. A.SSE.B.3.A
    b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. A.SSE.B.3.B
    c. Use the properties of exponents to transform expressions for exponential functions. A.SSE.B.3.C
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. A.SSE.B.4

Arithmetic with Polynomials and Rational Expressions

A. Perform arithmetic operations on polynomials. A.APR.A

1. Demonstrate understanding that polynomials form a system analogous to the integers; namely, they are closed under certain operations. A.APR.A.1

   a. Perform operations on polynomial expressions (addition, subtraction, multiplication, division) and compare the system of polynomials to the system of integers when performing operations. A.APR.A.1.A

   b. Factor and/or expand polynomial expressions, identify and combine like terms, and apply the distributive property. A.APR.A.1.B

B. Understand the relationship between zeros and factors of polynomials. A.APR.B

2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x - a is p(a), so p(a) = 0 if and only if (x - a) is a factor of p(x). A.APR.B.2

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. A.APR.B.3

C. Use polynomial identities to solve problems. A.APR.C

4. Prove polynomial identities and use them to describe numerical relationships. A.APR.C.4

5. (+) Know and apply the Binomial Theorem for the expansion of (x + y) ⁿ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined, for example, by Pascal's Triangle. A.APR.C.5

D. Rewrite rational expressions. A.APR.D

6. Rewrite simple rational expressions in different forms using inspection, long division, or, for the more complicated examples, a computer algebra system. A.APR.D.6

7. (+) Demonstrate understanding that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. A.APR.D.7

Creating Equations

A. Create equations that describe numbers or relationships. A.CED.A

1. Create one-variable equations and inequalities to solve problems, including linear, quadratic, rational, and exponential functions. A.CED.A.1
2. Interpret the relationship between two or more quantities. A.CED.A.2
    a. Define variables to represent the quantities and write equations to show the relationship. A.CED.A.2.A
    b. Use graphs to show a visual representation of the relationship while adhering to appropriate labels and scales. A.CED.A.2.B
3. Represent constraints using equations or inequalities and interpret solutions as viable or non-viable options in a modeling context. A.CED.A.3
4. Represent constraints using systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context. A.CED.A.4
5. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. A.CED.A.5

Reasoning with Equations and Inequalities

A. Understand solving equations as a process of reasoning and explain the reasoning. A.REI.A

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method. A.REI.A.1
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. A.REI.A.2

B. Solve equations and inequalities in one variable. A.REI.B

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. A.REI.B.3
   a. Solve linear equations and inequalities in one variable involving absolute value. A.REI.B.3.A
4. Solve quadratic equations in one variable. A.REI.B.4
   a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. A.REI.B.4.A
   b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. A.REI.B.4.B

C. Solve systems of equations. A.REI.C

5. Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. A.REI.C.5
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. A.REI.C.6
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. A.REI.C.7
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable. A.REI.C.8
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater). A.REI.C.9

D. Represent and solve equations and inequalities graphically. A.REI.D

10. Demonstrate understanding that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Show that any point on the graph of an equation in two variables is a solution to the equation. A.REI.D.10

11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. A.REI.D.11
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. A.REI.D.12

Functions (9-12)

Interpreting Functions

A. Understand the concept of a function and use function notation. F.IF.A

1. Demonstrate understanding that a function is a correspondence from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range: If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). F.IF.A.1
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F.IF.A.2
3. Demonstrate that a sequence is a functions, sometimes defined recursively, whose domain is a subset of the integers. F.IF.A.3

B. Interpret functions that arise in applications in terms of the context. Include linear, quadratic, exponential, rational, polynomial, square root and cube root, trigonometric, and logarithmic functions. F.IF.B

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior; and periodicity. F.IF.B.4
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.IF.B.5
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. F.IF.B.6

C. Analyze functions using different representations. F.IF.C

7. Graph functions expressed symbolically and show key features of the graphs, by hand in simple cases and using technology for more complicated cases. F.IF.C.7
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima. F.IF.C.7.A
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. F.IF.C.7.B
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. F.IF.C.7.C
   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. F.IF.C.7.D
   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.C.7.E
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.C.8

a. Use the process of factoring and/or completing the square in quadratic and polynomial functions, where appropriate, to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. F.IF.C.8.A

b. Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as identifying appreciation and depreciation rate for the value of a house or car sometime after its initial purchase. F.IF.C.8.B

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). F.IF.C.9

10. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric. F.IF.C.10

Building Functions

A. Build a function that models a relationship between two quantities. F.BF.A

1. Write a function that describes a relationship between two quantities. Functions could include linear, exponential, quadratic, simple rational, radical, logarithmic, and trigonometric. F.BF.A.1

    a. Determine an explicit expression, a recursive process, or steps for calculation from a context. F.BF.A.1.A

    b. Combine standard function types using arithmetic operations. F.BF.A.1.B

    c. (+) Compose functions. F.BF.A.1.C

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F.BF.A.2

B. Build new functions from existing functions. F.BF.B

3. Identify the effect on the graph of replacing f(x) by f(x) + k, ,f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Include, linear, quadratic, exponential, absolute value, simple rational and radical, logarithmic, and trigonometric functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph. Include recognizing even and odd functions from their graphs and algebraic expressions for them. F.BF.B.3

4. Find inverse functions algebraically and graphically. F.BF.B.4

    a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. Include linear and simple polynomial, rational, and exponential functions. F.BF.B.4.A

    b. (+) Verify by composition that one function is the inverse of another. F.BF.B.4.B

    c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. F.BF.B.4.C

    d. (+) Produce an invertible function from a non-invertible function by restricting the domain. F.BF.B.4.D

Linear, Quadratic, and Exponential Models

A. Construct and compare linear, quadratic, and exponential models and solve problems. F.LE.A

   1. Distinguish between situations that can be modeled with linear functions and with exponential functions. F.LE.A.1

      a. Demonstrate that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. F.LE.A.1.A

      b. Identify situations in which one quantity changes at a constant rate per unit interval relative to another. F.LE.A.1.B

      c. Identify situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F.LE.A.1.C

   2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table). F.LE.A.2

   3. Use graphs and tables to demonstrate that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. F.LE.A.3

   4. For exponential models, express as a logarithm the solution to ab^{ct} = d where a, c, and d are numbers and the base b is 2,10, or e; evaluate the logarithm using technology. F.LE.A.4

B. Interpret expressions for functions in terms of the situation they model. F.LE.B

   5. Interpret the parameters in a linear or exponential function (of the form f(x) = b^x + k) in terms of a context. F.LE.B.5

Trigonometric Functions

A. Extend the domain of trigonometric functions using the unit circle. F.TF.A

1. Demonstrate radian measure as the ratio of the arc length subtended by a central angle to the length of the radius of the unit circle. F.TF.A.1

   a. Use radian measure to solve problems. F.TF.A.1.A

2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. F.TF.A.2

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. F.TF.A.3

4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. F.TF.A.4

B. Model periodic phenomena with trigonometric functions. F.TF.B

5. Model periodic phenomena using trigonometric functions with specified amplitude, frequency, and midline. F.TF.B.5

6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. F.TF.B.6

7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. F.TF.B.7

C. Prove and apply trigonometric identities. F.TF.C

8. Relate the Pythagorean Theorem to the unit circle to discover the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use the Pythagorean identity to find the value of a trigonometric function ($\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$) given one trigonometric function ($\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$) and the quadrant of the angle. F.TF.C.8

9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. F.TF.C.9

Geometry (9-12)

Congruence

A. Experiment with transformations in the plane. G.CO.A

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. G.CO.A.1
2. Represent transformations in the plane and describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not. G.CO.A.2
3. Describe the rotations and reflections that carry a given figure (rectangle, parallelogram, trapezoid, or regular polygon) onto itself. G.CO.A.3
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. G.CO.A.4
5. Draw the transformation (rotation, reflection, or translation) for a given geometric figure. G.CO.A.5
6. Specify a sequence of transformations that will carry a given figure onto another. G.CO.A.6

B. Understand congruence in terms of rigid motions. G.CO.B

7. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G.CO.B.7
8. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G.CO.B.8
9. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. G.CO.B.9

C. Prove geometric theorems and, when appropriate, the converse of theorems. G.CO.C

10. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. G.CO.C.10
11. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. G.CO.C.11
12. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. G.CO.C.12

a. Prove theorems about polygons. Theorems include: the measures of interior and exterior angles; apply properties of polygons to the solutions of mathematical and contextual problems. G.CO.C.12.A

D. Make geometric constructions. G.CO.D

13. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.) Constructions include: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. G.CO.D.13

14. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. G.CO.D.14

Similarity, Right Triangles, and Trigonometry

A. Understand similarity in terms of similarity transformations. G.SRT.A

1. Verify experimentally the properties of dilations given by a center and a scale factor. G.SRT.A.1
   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. G.SRT.A.1.A
   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. G.SRT.A.1.B
2. Use the definition of similarity to decide if two given figures are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. G.SRT.A.2
3. Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar. G.SRT.A.3

B. Prove theorems involving similarity. G.SRT.B

4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. G.SRT.B.4
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. G.SRT.B.5

C. Define trigonometric ratios and solve problems involving right triangles. G.SRT.C

6. Demonstrate understanding that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G.SRT.C.6
7. Explain and use the relationship between the sine and cosine of complementary angles. G.SRT.C.7
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. G.SRT.C.8

D. Apply trigonometry to general triangles. G.SRT.D

9. (+) Derive the formula A = ½ absin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. G.SRT.D.9
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems. G.SRT.D.10
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems; resultant forces). G.SRT.D.11

Circles

A. Understand and apply theorems about circles. G.C.A

1. Prove that all circles are similar. G.C.A.1
2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. G.C.A.2
3. Prove properties of angles for a quadrilateral and other polygons inscribed in a circle, by constructing the inscribed and circumscribed circles of a triangle. G.C.A.3
4. (+) Construct a tangent line to a circle from a point outside the given circle. G.C.A.4

B. Find arc lengths and areas of sectors of circles. G.C.B

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. G.C.B.5

Expressing Geometric Properties with Equations

A. Translate between the geometric description and the equation for a conic section. G.GPE.A

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. G.GPE.A.1
2. Derive the equation of a parabola given a focus and directrix. G.GPE.A.2
3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. G.GPE.A.3
    a. (+) Use equations and graphs of conic sections to model real-world problems. G.GPE.A.3.A

B. Use coordinates to prove simple geometric theorems algebraically. G.GPE.B

4. Use coordinates to prove simple geometric theorems algebraically, including the distance formula and its relationship to the Pythagorean Theorem. G.GPE.B.4
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems. G.GPE.B.5
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. G.GPE.B.6
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula). G.GPE.B.7

Geometric Measurement and Dimension

A. Explain volume formulas and use them to solve problems. G.GMD.A

1. Give an informal argument for the formulas for the circumference of a circle; area of a circle; volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. G.GMD.A.1
2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. G.GMD.A.2
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. G.GMD.A.3

B. Visualize relationships between two-dimensional and three-dimensional objects. G.GMD.B

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. G.GMD.B.4

Modeling with Geometry

A. Apply geometric concepts in modeling situations. G.MG.A

1. Use geometric shapes, their measures, and their properties to describe objects. G.MG.A.1
2. Apply concepts of density based on area and volume in modeling situations. G.MG.A.2
3. Apply geometric methods to solve design problems. G.MG.A.3
4. Use dimensional analysis for unit conversions to confirm that expressions and equations make sense. G.MG.A.4

Statistics and Probability (9-12)

Interpreting Categorical and Quantitative Data

A. Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate. S.ID.A

1. Differentiate between count data and measurement variable. S.ID.A.1
2. Represent measurement data with plots on the real number line (dot plots, histograms, and box plots). S.ID.A.2
3. Compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different variables, using statistics appropriate to the shape of the distribution for each measurement variable. S.ID.A.3
4. Interpret differences in shape, center, and spread in the context of the variables accounting for possible effects of extreme data points (outliers) for measurement variables. S.ID.A.4
5. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. S.ID.A.5

B. Summarize, represent, and interpret data on two categorical and quantitative variables. S.ID.B

6. Represent data on two categorical variables on a clustered bar chart and describe how the variables are related. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. S.ID.B.6
7. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. S.ID.B.7
    a. Fit a linear function to data where a scatter plot suggests a linear relationship and use the fitted function to solve problems in the context of the data. S.ID.B.7.A
    b. Use functions fitted to data, focusing on quadratic and exponential models, or choose a function suggested by the context. Utilize technology where appropriate. S.ID.B.7.B
    c. Informally assess the fit of a function by plotting and analyzing residuals. S.ID.B.7.C

C. Interpret linear models. S.ID.C

8. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. S.ID.C.8
9. Compute (using technology) and interpret the linear correlation coefficient. S.ID.C.9
10. Distinguish between (linear) correlation and causation. S.ID.C.10

Making Inferences and Justifying Conclusions

A. Understand and evaluate random processes underlying statistical studies. Use calculators, spreadsheets, and other technology as appropriate. S.IC.A

   1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. S.IC.A.1

   2. Decide if a specified model is consistent with results from a given data-generating process (e.g., using simulation or validation with given data). S.IC.A.2

B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies. S.IC.B

   3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S.IC.B.3

   4. Use data from a sample survey to estimate a population mean or proportion and a margin of error. S.IC.B.4

   5. Use data from a randomized and controlled experiment to compare two treatments; use margins of error to decide if differences between treatments are significant. S.IC.B.5

   6. Evaluate reports of statistical information based on data. S.IC.B.6

Conditional Probability and the Rules of Probability

A. Understand independence and conditional probability and use them to interpret data from simulations or experiments. S.CP.A

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). S.CP.A.1
2. Demonstrate understanding that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S.CP.A.2
3. Understand the conditional probability of A given B as $P(A \cap B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. S.CP.A.3
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. S.CP.A.4
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. S.CP.A.5

B. Use the rules of probability to compute probabilities of compound events in a uniform probability model. S.CP.B

6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. S.CP.B.6
7. Apply the Addition Rule, $P(A \cup B) = P(A) + P(B) - P(A \cup B)$, and interpret the answer in terms of the model. S.CP.B.7
8. (+) Apply the general Multiplication Rule in a uniform probability model $P(A \cap B) = P(A)P(B/A) = P(B)P(A/B)$, and interpret the answer in terms of the model. S.CP.B.8
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. S.CP.B.9

Using Probability to Make Decisions

A. Calculate expected values and use them to solve problems. S.MD.A

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. S.MD.A.1
2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution of the variable. S.MD.A.2
3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. S.MD.A.3
4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. S.MD.A.4

B. Use probability to evaluate outcomes of decisions. S.MD.B

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. S.MD.B.5
   a. Find the expected payoff for a game of chance. S.MD.B.5.A
   b. Evaluate and compare strategies on the basis of expected values. S.MD.B.5.B
6. (+) Use probabilities to make objective decisions. S.MD.B.6
7. (+) Analyze decisions and strategies using probability concepts. S.MD.B.7