Georgia Mathematics

# Differential Equations

## Mathematical Practices

**0 Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.** DE.MP

- 0.1 Make sense of problems and persevere in solving them. DE.MP.1
- 0.2 Reason abstractly and quantitatively. DE.MP.2
- 0.3 Construct viable arguments and critique the reasoning of others. DE.MP.3
- 0.4 Model with mathematics. DE.MP.4
- 0.5 Use appropriate tools strategically. DE.MP.5
- 0.6 Attend to precision. DE.MP.6
- 0.7 Look for and make use of structure. DE.MP.7
- 0.8 Look for and express regularity in repeated reasoning. DE.MP.8

## Mathematical Modeling

**1 Apply mathematics to real-life situations; model real-life phenomena using mathematics.** DE.MM.1

- 1.1 Explain contextual, mathematical problems using a mathematical model. DE.MM.1.1
- 1.2 Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts. DE.MM.1.2
- 1.3 Using abstract and quantitative reasoning, make decisions about information and data from a contextual situation. DE.MM.1.3
- 1.4 Use various mathematical representations and structures with this information to represent and solve real-life problems. DE.MM.1.4

**Abstract Reasoning**

2 **Solve contextual, mathematical problems involving first-order differential equations to explain real-life phenomena.** DE.AR.2

- 2.1 Classify differential equations by order and linearity. DE.AR.2.1
- 2.2 Solve separable differential equations for general solutions and initial value problems. DE.AR.2.2
- 2.3 Solve first-order linear differential equations and initial value problems using integrating factors. DE.AR.2.3
- 2.4 Use modeling or numerical methods to approximate solutions of first-order differential equations in context. DE.AR.2.4
- 2.5 Draw direction fields containing solutions curves for first-order differential equations by hand and using modeling. DE.AR.2.5
- 2.6 Solve first-order linear differential equations that apply to various real-world models including falling bodies, mixtures, population and the logistic equation, continuously compounded interest, and other physics applications. DE.AR.2.6

**3 Solve contextual, mathematical problems involving second and higher order differential equations to explain real-life phenomena.** DE.AR.3

- 3.1 Determine whether a first- or second-order differential equation has a unique solution over a given interval by applying the Existence and Uniqueness Theorem. DE.AR.3.1
- 3.2 Solve second-order linear homogeneous and non-homogeneous differential equations by finding characteristic equations, using the method of undetermined coefficients and variation of parameters. DE.AR.3.2
- 3.3 Solve second-order differential equations that apply to various real-world models. DE.AR.3.3
- 3.4 Use vector function notation when discussing the structure of solution sets for homogeneous systems as it pertains to the Wronskian. DE.AR.3.4
- 3.5 Determine the existence and uniqueness of solutions for second-order linear differential equations, determine a fundamental set of solutions, and verify that two solutions form a fundamental set by taking the Wronskian. DE.AR.3.5
- 3.6 Determine the structure of solution set to higher-order differential equations, apply the basic Existence and Uniqueness Theorem to higher-order differential equations, and use the generalizations of the Wronksian for higher order differential equations. DE.AR.3.6
- 3.7 Solve higher-order constant coefficient homogeneous differential equations. DE.AR.3.7
- 3.8 Solve special case non-homogeneous second order ordinary differential equations including Cauchy-Euler Equations. DE.AR.3.8
- 3.9 Find a second linearly dependent solution using reduction of order when given a solution to a non-homogeneous second-order differential equation. DE.AR.3.9
- 3.10 Determine ordinary points, recurrence relations, and change the index as they relate to series solutions to ordinary differential equations. DE.AR.3.10
- 3.11 Find series solutions to first and second-order non-linear initial value problems. DE.AR.3.11

**4 Solve contextual, mathematical problems involving systems of differential equations to explain real-life phenomena.** DE.AR.4

- 4.1 Determine whether a contextual situation results in a system of differential equations and apply the basic existence and uniqueness results for the corresponding initial value problems. DE.AR.4.1
- 4.2 Solve constant coefficient homogeneous systems using eigenvalues and eigenvectors. Solve systems with real, distinct eigenvalues, as well as those with repeated and imaginary eigenvalues. DE.AR.4.2
- 4.3 Draw phase portraits for solutions of homogeneous systems with constant coefficients. DE.AR.4.3
- 4.4 Solve non-homogeneous systems of ordinary differential equations using the method of undetermined coefficients and variation of parameters. DE.AR.4.4
- 4.5 Determine which non-linear systems are locally linear and identify the behavior of the system about each critical point. DE.AR.4.5
- 4.6 Plot locally linear systems. DE.AR.4.6
- 4.7 Use population models derived from locally linear systems. DE.AR.4.7

**5 Solve contextual, mathematical problems using Laplace transforms to explain real-life phenomena.** DE.AR.5

- 5.1 Use the integral definition to perform Laplace transforms for functions. DE.AR.5.1
- 5.2 Use a Laplace table to accurately and efficiently identify Laplace transforms. DE.AR.5.2
- 5.3 Perform inverse Laplace transforms using a variety of techniques. DE.AR.5.3
- 5.4 Solve first- and second-order differential equations using Laplace transforms that apply to fields such as electrical and mechanical engineering. DE.AR.5.4
- 5.5 Write piecewise functions as compositions of step (Heaviside) functions. DE.AR.5.5
- 5.6 Find the general uniqueness and existence of solutions for step functions, and use Laplace transforms to find solutions to step functions. DE.AR.5.6
- 5.7 Find the Laplace transform of the Dirac delta function. DE.AR.5.7
- 5.8 Solve linear systems of differential equations using Laplace transforms. DE.AR.5.8