Alabama Mathematics

Grades 9, 10, 11, 12 (All Courses)

Adopted 2019

Student Mathematical Practices

1. **Make sense of problems and persevere in solving them.** MP.1
2. **Reason abstractly and quantitatively.** MP.2
3. **Construct viable arguments and critique the reasoning of others.** MP.3
4. **Model with mathematics.** MP.4
5. **Use appropriate tools strategically.** MP.5
6. **Attend to precision.** MP.6
7. **Look for and make use of structure.** MP.7
8. **Look for and express regularity in repeated reasoning.** MP.8

Geometry with Data Analysis

Number and Quantity

A. Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers. GDA.NQ.A

1. Extend understanding of irrational and rational numbers by rewriting expressions involving radicals, including addition, subtraction, multiplication, and division, in order to recognize geometric patterns. GDA.NQ.A.1

B. Quantitative reasoning includes and mathematical modeling requires attention to units of measurement. GDA.NQ.B

2. Use units as a way to understand problems and to guide the solution of multi-step problems. GDA.NQ.B.2

    a. Choose and interpret units consistently in formulas. GDA.NQ.B.2.A

    b. Choose and interpret the scale and the origin in graphs and data displays. GDA.NQ.B.2.B

    c. Define appropriate quantities for the purpose of descriptive modeling. GDA.NQ.B.2.C

    d. Choose a level of accuracy appropriate to limitations of measurements when reporting quantities. GDA.NQ.B.2.D

Algebra and Functions

A. The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution. GDA.AF.A

   3. Find the coordinates of the vertices of a polygon determined by a set of lines, given their equations, by setting their function rules equal and solving, or by using their graphs. GDA.AF.A.3

B. Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts – in particular, contexts that arise in relation to linear, quadratic, and exponential situations. GDA.AF.B

   4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. GDA.AF.B.4

C. Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities—including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology). GDA.AF.C

   5. Verify that the graph of a linear equation in two variables is the set of all its solutions plotted in the coordinate plane, which forms a line. GDA.AF.C.5

   6. Derive the equation of a circle of given center and radius using the Pythagorean Theorem. GDA.AF.C.6

      a. Given the endpoints of the diameter of a circle, use the midpoint formula to find its center and then use the Pythagorean Theorem to find its equation. GDA.AF.C.6.A

      b. Derive the distance formula from the Pythagorean Theorem. GDA.AF.C.6.B

Data Analysis, Statistics, and Probability

A. Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks. GDA.DSP.A

   7. Use mathematical and statistical reasoning with quantitative data, both univariate data (set of values) and bivariate data (set of pairs of values) that suggest a linear association, in order to draw conclusions and assess risk. GDA.DSP.A.7

B. Data arise from a context and come in two types: quantitative (continuous or discrete) and categorical. Technology can be used to "clean" and organize data, including very large data sets, into a useful and manageable structure – a first step in any analysis of data GDA.DSP.B

   8. Use technology to organize data, including very large data sets, into a useful and manageable structure. GDA.DSP.B.8

C. Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable. GDA.DSP.C

   9. Represent the distribution of univariate quantitative data with plots on the real number line, choosing a format (dot plot, histogram, or box plot) most appropriate to the data set, and represent the distribution of bivariate quantitative data with a scatter plot. Extend from simple cases by hand to more complex cases involving large data sets using technology. GDA.DSP.C.9

   10. Use statistics appropriate to the shape of the data distribution to compare and contrast two or more data sets, utilizing the mean and median for center and the interquartile range and standard deviation for variability. GDA.DSP.C.10

       a. Explain how standard deviation develops from mean absolute deviation. GDA.DSP.C.10.A

       b. Calculate the standard deviation for a data set, using technology where appropriate. GDA.DSP.C.10.B

   11. Interpret differences in shape, center, and spread in the context of data sets, accounting for possible effects of extreme data points (outliers) on mean and standard deviation. GDA.DSP.C.11

D. Scatter plots, including plots over time, can reveal patterns, trends, clusters, and gaps that are useful in analyzing the association between two contextual variables. GDA.DSP.D

   12. Represent data of two quantitative variables on a scatter plot, and describe how the variables are related. GDA.DSP.D.12

       a. Find a linear function for a scatter plot that suggests a linear association and informally assess its fit by plotting and analyzing residuals, including the squares of the residuals, in order to improve its fit. GDA.DSP.D.12.A

       b. Use technology to find the least-squares line of best fit for two quantitative variables. GDA.DSP.D.12.B

E. Analyzing the association between two quantitative variables should involve statistical procedures, such as examining (with technology) the sum of squared deviations in fitting a linear model, analyzing residuals for patterns, generating a least-squares regression line and finding a correlation coefficient, and differentiating between correlation and causation. `GDA.DSP.E`

13. Compute (using technology) and interpret the correlation coefficient of a linear relationship. `GDA.DSP.E.13`

14. Distinguish between correlation and causation. `GDA.DSP.E.14`

F. Data analysis techniques can be used to develop models of contextual situations and to generate and evaluate possible solutions to real problems involving those contexts. `GDA.DSP.F`

15. Evaluate possible solutions to real-life problems by developing linear models of contextual situations and using them to predict unknown values. `GDA.DSP.F.15`

    a. Use the linear model to solve problems in the context of the given data. `GDA.DSP.F.15.A`

    b. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the given data. `GDA.DSP.F.15.B`

Geometry and Measurement

A. Areas and volumes of figures can be computed by determining how the figure might be obtained from simpler figures by dissection and recombination. GDA.GM.A

  16. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. GDA.GM.A.16

  17. Model and solve problems using surface area and volume of solids, including composite solids and solids with portions removed. GDA.GM.A.17

    a. Give an informal argument for the formulas for the surface area and volume of a sphere, cylinder, pyramid, and cone using dissection arguments, Cavalieri's Principle, and informal limit arguments. GDA.GM.A.17.A

    b. Apply geometric concepts to find missing dimensions to solve surface area or volume problems. GDA.GM.A.17.B

B. Constructing approximations of measurements with different tools, including technology, can support an understanding of measurement. GDA.GM.B

  18. Given the coordinates of the vertices of a polygon, compute its perimeter and area using a variety of methods, including the distance formula and dynamic geometry software, and evaluate the accuracy of the results. GDA.GM.B.18

C. When an object is the image of a known object under a similarity transformation, a length, area, or volume on the image can be computed by using proportional relationships. GDA.GM.C

  19. Derive and apply the relationships between the lengths, perimeters, areas, and volumes of similar figures in relation to their scale factor. GDA.GM.C.19

  20. Derive and apply the formula for the length of an arc and the formula for the area of a sector. GDA.GM.C.20

D. Applying geometric transformations to figures provides opportunities for describing the attributes of the figures preserved by the transformation and for describing symmetries by examining when a figure can be mapped onto itself. GDA.GM.D

  21. Represent transformations and compositions of transformations in the plane (coordinate and otherwise) using tools such as tracing paper and geometry software. GDA.GM.D.21

    a. Describe transformations and compositions of transformations as functions that take points in the plane as inputs and give other points as outputs, using informal and formal notation. GDA.GM.D.21.A

    b. Compare transformations which preserve distance and angle measure to those that do not. GDA.GM.D.21.B

  22. Explore rotations, reflections, and translations using graph paper, tracing paper, and geometry software. GDA.GM.D.22

    a. Given a geometric figure and a rotation, reflection, or translation, draw the image of the transformed figure using graph paper, tracing paper, or geometry software. GDA.GM.D.22.A

b. Specify a sequence of rotations, reflections, or translations that will carry a given figure onto another. GDA.GM.D.22.B

c. Draw figures with different types of symmetries and describe their attributes. GDA.GM.D.22.C

23. Develop definitions of rotation, reflection, and translation in terms of angles, circles, perpendicular lines, parallel lines, and line segments. GDA.GM.D.23

E. Showing that two figures are congruent involves showing that there is a rigid motion (translation, rotation, reflection, or glide reflection) or, equivalently, a sequence of rigid motions that maps one figure to the other. GDA.GM.E

24. Define congruence of two figures in terms of rigid motions (a sequence of translations, rotations, and reflections); show that two figures are congruent by finding a sequence of rigid motions that maps one figure to the other. GDA.GM.E.24

25. Verify criteria for showing triangles are congruent using a sequence of rigid motions that map one triangle to another. GDA.GM.E.25

a. Verify that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. GDA.GM.E.25.A

b. Verify that two triangles are congruent if (but not only if) the following groups of corresponding parts are congruent: angle-side-angle (ASA), side-angle-side (SAS), side-side-side (SSS), and angle-angle-side (AAS). GDA.GM.E.25.B

F. Showing that two figures are similar involves finding a similarity transformation (dilation or composite of a dilation with a rigid motion) or, equivalently, a sequence of similarity transformations that maps one figure onto the other. GDA.GM.F

26. Verify experimentally the properties of dilations given by a center and a scale factor. GDA.GM.F.26

a. Verify that a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. GDA.GM.F.26.A

b. Verify that the dilation of a line segment is longer or shorter in the ratio given by the scale factor. GDA.GM.F.26.B

27. Given two figures, determine whether they are similar by identifying a similarity transformation (sequence of rigid motions and dilations) that maps one figure to the other. GDA.GM.F.27

28. Verify criteria for showing triangles are similar using a similarity transformation (sequence of rigid motions and dilations) that maps one triangle to another. GDA.GM.F.28

a. Verify that two triangles are similar if and only if corresponding pairs of sides are proportional and corresponding pairs of angles are congruent. GDA.GM.F.28.A

b. Verify that two triangles are similar if (but not only if) two pairs of corresponding angles are congruent (AA), the corresponding sides are proportional (SSS), or two pairs of corresponding sides are proportional and the pair of included angles is congruent (SAS). GDA.GM.F.28.B

G. Using technology to construct and explore figures with constraints provides an opportunity to explore the independence and dependence of assumptions and conjectures. GDA.GM.G

29. Find patterns and relationships in figures including lines, triangles, quadrilaterals, and circles, using technology and other tools. GDA.GM.G.29

a. Construct figures, using technology and other tools, in order to make and test conjectures about their properties. GDA.GM.G.29.A

b. Identify different sets of properties necessary to define and construct figures. GDA.GM.G.29.B

H. Proof is the means by which we demonstrate whether a statement is true or false mathematically, and proofs can be communicated in a variety of ways (e.g., two-column, paragraph). GDA.GM.H

30. Develop and use precise definitions of figures such as angle, circle, perpendicular lines, parallel lines, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. GDA.GM.H.30

31. Justify whether conjectures are true or false in order to prove theorems and then apply those theorems in solving problems, communicating proofs in a variety of ways, including flow chart, two-column, and paragraph formats. GDA.GM.H.31

a. Investigate, prove, and apply theorems about lines and angles, including but not limited to: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; the points on the perpendicular bisector of a line segment are those equidistant from the segment's endpoints. GDA.GM.H.31.A

b. Investigate, prove, and apply theorems about triangles, including but not limited to: the sum of the measures of the interior angles of a triangle is 180°; the base angles of isosceles triangles are congruent; the segment joining the midpoints of two sides of a triangle is parallel to the third side and half the length; a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem using triangle similarity. GDA.GM.H.31.B

c. Investigate, prove, and apply theorems about parallelograms and other quadrilaterals, including but not limited to both necessary and sufficient conditions for parallelograms and other quadrilaterals, as well as relationships among kinds of quadrilaterals. GDA.GM.H.31.C

I. Proofs of theorems can sometimes be made with transformations, coordinates, or algebra; all approaches can be useful, and in some cases one may provide a more accessible or understandable argument than another. GDA.GM.I

32. Use coordinates to prove simple geometric theorems algebraically. GDA.GM.I.32

33. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems. `GDA.GM.I.33`

J. Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry, in real-world contexts provides a means of building understanding of these concepts and is a powerful tool for solving problems related to the physical world in which we live. `GDA.GM.J`

34. Use congruence and similarity criteria for triangles to solve problems in real-world contexts. `GDA.GM.J.34`

35. Discover and apply relationships in similar right triangles. `GDA.GM.J.35`

    a. Derive and apply the constant ratios of the sides in special right triangles (45˚-45˚-90˚ and 30˚-60˚-90˚). `GDA.GM.J.35.A`

    b. Use similarity to explore and define basic trigonometric ratios, including sine ratio, cosine ratio, and tangent ratio. `GDA.GM.J.35.B`

    c. Explain and use the relationship between the sine and cosine of complementary angles. `GDA.GM.J.35.C`

    d. Demonstrate the converse of the Pythagorean Theorem. `GDA.GM.J.35.D`

    e. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems, including finding areas of regular polygons. `GDA.GM.J.35.E`

36. Use geometric shapes, their measures, and their properties to model objects and use those models to solve problems. `GDA.GM.J.36`

37. Investigate and apply relationships among inscribed angles, radii, and chords, including but not limited to: the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. `GDA.GM.J.37`

K. Experiencing the mathematical modeling cycle in problems involving geometric concepts, from the simplification of the real problem through the solving of the simplified problem, the interpretation of its solution, and the checking of the solution's feasibility, introduces geometric techniques, tools, and points of view that are valuable to problem-solving. `GDA.GM.K`

38. Use the mathematical modeling cycle involving geometric methods to solve design problems. `GDA.GM.K.38`

Algebra I With Probability

Number and Quantity

A. Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers. A1P.NQ.A

1. Explain how the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for an additional notation for radicals using rational exponents. A1P.NQ.A.1
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. A1P.NQ.A.2
3. Define the imaginary number i such that i^2 = -1. A1P.NQ.A.3

Algebra and Functions

A. Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible. A1P.AF.A

4. Interpret linear, quadratic, and exponential expressions in terms of a context by viewing one or more of their parts as a single entity. A1P.AF.A.4

5. Use the structure of an expression to identify ways to rewrite it. A1P.AF.A.5

6. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A1P.AF.A.6

   a. Factor quadratic expressions with leading coefficients of one, and use the factored form to reveal the zeros of the function it defines. A1P.AF.A.6.A

   b. Use the vertex form of a quadratic expression to reveal the maximum or minimum value and the axis of symmetry of the function it defines; complete the square to find the vertex form of quadratics with a leading coefficient of one. A1P.AF.A.6.B

   c. Use the properties of exponents to transform expressions for exponential functions. A1P.AF.A.6.C

7. Add, subtract, and multiply polynomials, showing that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication. A1P.AF.A.7

B. Finding solutions to an equation, inequality, or system of equations or inequalities requires the checking of candidate solutions, whether generated analytically or graphically, to ensure that solutions are found and that those found are not extraneous. A1P.AF.B

8. Explain why extraneous solutions to an equation involving absolute values may arise and how to check to be sure that a candidate solution satisfies an equation. A1P.AF.B.8

C. The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution. A1P.AF.C

9. Select an appropriate method to solve a quadratic equation in one variable. A1P.AF.C.9

   a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)^2 = q that has the same solutions. Explain how the quadratic formula is derived from this form. A1P.AF.C.9.A

   b. Solve quadratic equations by inspection (such as x^2 = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation, and recognize that some solutions may not be real. A1P.AF.C.9.B

10. Select an appropriate method to solve a system of two linear equations in two variables. A1P.AF.C.10

    a. Solve a system of two equations in two variables by using linear combinations; contrast situations in which use of linear combinations is more efficient with those in which substitution is more efficient. A1P.AF.C.10.A

    b. Contrast solutions to a system of two linear equations in two variables produced by algebraic methods with graphical and tabular methods. A1P.AF.C.10.B

D. Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts – in particular, contexts that arise in relation to linear, quadratic, and exponential situations. A1P.AF.D

11. Create equations and inequalities in one variable and use them to solve problems in context, either exactly or approximately. Extend from contexts arising from linear functions to those involving quadratic, exponential, and absolute value functions. A1P.AF.D.11

12. Create equations in two or more variables to represent relationships between quantities in context; graph equations on coordinate axes with labels and scales and use them to make predictions. Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions. A1P.AF.D.12

13. Represent constraints by equations and/or inequalities, and solve systems of equations and/or inequalities, interpreting solutions as viable or nonviable options in a modeling context. Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions. A1P.AF.D.13

E. Functions shift the emphasis from a point-by-point relationship between two variables (input/output) to considering an entire set of ordered pairs (where each first element is paired with exactly one second element) as an entity with its own features and characteristics. A1P.AF.E

14. Given a relation defined by an equation in two variables, identify the graph of the relation as the set of all its solutions plotted in the coordinate plane. A1P.AF.E.14

15. Define a function as a mapping from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range. A1P.AF.E.15

    a. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. A1P.AF.E.15.A

    b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Limit to linear, quadratic, exponential, and absolute value functions. A1P.AF.E.15.B

16. Compare and contrast relations and functions represented by equations, graphs, or tables that show related values; determine whether a relation is a

function. Explain that a function f is a special kind of relation defined by the equation y = f(x). A1P.AF.E.16

17. Combine different types of standard functions to write, evaluate, and interpret functions in context. Limit to linear, quadratic, exponential, and absolute value functions. A1P.AF.E.17

    a. Use arithmetic operations to combine different types of standard functions to write and evaluate functions. A1P.AF.E.17.A

    b. Use function composition to combine different types of standard functions to write and evaluate functions. A1P.AF.E.17.B

F. Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities – including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology). A1P.AF.F

18. Solve systems consisting of linear and/or quadratic equations in two variables graphically, using technology where appropriate. A1P.AF.F.18

19. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x). A1P.AF.F.19

    a. Find the approximate solutions of an equation graphically, using tables of values, or finding successive approximations, using technology where appropriate. Note: Include cases where f(x) is a linear, quadratic, exponential, or absolute value function and g(x) is constant or linear. A1P.AF.F.19.A

20. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes, using technology where appropriate. A1P.AF.F.20

G. Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g., f(x) = x^2), recursive definitions, tables, and graphs. A1P.AF.G

21. Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend from linear to quadratic, exponential, absolute value, and general piecewise. A1P.AF.G.21

22. Define sequences as functions, including recursive definitions, whose domain is a subset of the integers. A1P.AF.G.22

    a. Write explicit and recursive formulas for arithmetic and geometric sequences and connect them to linear and exponential functions. A1P.AF.G.22.A

H. Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family. A1P.AF.H

23. Identify the effect on the graph of replacing f(x) by f(x) + k, k·f(x), f(f·x), for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and explain the effects on the graph, using technology as appropriate. Limit to linear, quadratic, exponential, absolute value, and linear piecewise functions. A1P.AF.H.23

24. Distinguish between situations that can be modeled with linear functions and those that can be modeled with exponential functions. A1P.AF.H.24

    a. Show that linear functions grow by equal differences over equal intervals, while exponential functions grow by equal factors over equal intervals. A1P.AF.H.24.A

    b. Define linear functions to represent situations in which one quantity changes at a constant rate per unit interval relative to another. A1P.AF.H.24.B

    c. Define exponential functions to represent situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. A1P.AF.H.24.C

25. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). A1P.AF.H.25

26. Use graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. A1P.AF.H.26

27. Interpret the parameters of functions in terms of a context. Extend from linear functions, written in the form mx + b, to exponential functions, written in the form ab^x. Functions can be represented graphically and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation. A1P.AF.H.27

28. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Extend from relationships that can be represented by linear functions to quadratic, exponential, absolute value, and linear piecewise functions. A1P.AF.H.28

29. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Limit to linear, quadratic, exponential, and absolute value functions. A1P.AF.H.29

30. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. A1P.AF.H.30

    a. Graph linear and quadratic functions and show intercepts, maxima, and minima. A1P.AF.H.30.A

    b. Graph piecewise-defined functions, including step functions and absolute value functions. A1P.AF.H.30.B

c. Graph exponential functions, showing intercepts and end behavior. A1P.AF.H.30.C

I. Functions model a wide variety of real situations and can help students understand the processes of making and changing assumptions, assigning variables, and finding solutions to contextual problems. A1P.AF.I

31. Use the mathematical modeling cycle to solve real-world problems involving linear, quadratic, exponential, absolute value, and linear piecewise functions. A1P.AF.I.31

Data Analysis, Statistics, and Probability

A. Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks. `A1P.DSP.A`

  A32. Use mathematical and statistical reasoning with bivariate categorical data in order to draw conclusions and assess risk. `A1P.DSP.A32`

B. Making and defending informed, data-based decisions is a characteristic of a quantitatively literate person. `A1P.DSP.B`

  33. Design and carry out an investigation to determine whether there appears to be an association between two categorical variables, and write a persuasive argument based on the results of the investigation. `A1P.DSP.B.33`

C. Data arise from a context and come in two types: quantitative (continuous or discrete) and categorical. Technology can be used to "clean" and organize data, including very large data sets, into a useful and manageable structure—a first step in any analysis of data. `A1P.DSP.C`

  34. Distinguish between quantitative and categorical data and between the techniques that may be used for analyzing data of these two types. `A1P.DSP.C.34`

D. The association between two categorical variables is typically represented by using two-way tables and segmented bar graphs. `A1P.DSP.D`

  35. Analyze the possible association between two categorical variables. `A1P.DSP.D.35`

    a. Summarize categorical data for two categories in two-way frequency tables and represent using segmented bar graphs. `A1P.DSP.D.35.A`

    b. Interpret relative frequencies in the context of categorical data (including joint, marginal, and conditional relative frequencies). `A1P.DSP.D.35.B`

    c. Identify possible associations and trends in categorical data. `A1P.DSP.D.35.C`

E. Data analysis techniques can be used to develop models of contextual situations and to generate and evaluate possible solutions to real problems involving those contexts. `A1P.DSP.E`

  36. Generate a two-way categorical table in order to find and evaluate solutions to real-world problems. `A1P.DSP.E.36`

    a. Aggregate data from several groups to find an overall association between two categorical variables. `A1P.DSP.E.36.A`

    b. Recognize and explore situations where the association between two categorical variables is reversed when a third variable is considered (Simpson's Paradox). `A1P.DSP.E.36.B`

F. Two events are independent if the occurrence of one event does not affect the probability of the other event. Determining whether two events are independent can be used for finding and understanding probabilities. `A1P.DSP.F`

37. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). `A1P.DSP.F.37`

38. Explain whether two events, A and B, are independent, using two-way tables or tree diagrams. `A1P.DSP.F.38`

G. Conditional probabilities – that is, those probabilities that are "conditioned" by some known information – can be computed from data organized in contingency tables. Conditions or assumptions may affect the computation of a probability. `A1P.DSP.G`

39. Compute the conditional probability of event A given event B, using two-way tables or tree diagrams. `A1P.DSP.G.39`

40. Recognize and describe the concepts of conditional probability and independence in everyday situations and explain them using everyday language. `A1P.DSP.G.40`

41. Explain why the conditional probability of A given B is the fraction of B's outcomes that also belong to A, and interpret the answer in context. `A1P.DSP.G.41`

Algebra II With Statistics

Number and Quantity

A. Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers. `A2S.NQ.A`

1. Identify numbers written in the form a + bi, where a and b are real numbers and $i^2 = -1$, as complex numbers. `A2S.NQ.A.1`

   a. Add, subtract, and multiply complex numbers using the commutative, associative, and distributive properties. `A2S.NQ.A.1.A`

B. Matrices are a useful way to represent information. `A2S.NQ.B`

2. Use matrices to represent and manipulate data. `A2S.NQ.B.2`

3. Multiply matrices by scalars to produce new matrices. `A2S.NQ.B.3`

4. Add, subtract, and multiply matrices of appropriate dimensions. `A2S.NQ.B.4`

5. Describe the roles that zero and identity matrices play in matrix addition and multiplication, recognizing that they are similar to the roles of 0 and 1 in the real numbers. `A2S.NQ.B.5`

   a. Find the additive and multiplicative inverses of square matrices, using technology as appropriate. `A2S.NQ.B.5.A`

   b. Explain the role of the determinant in determining if a square matrix has a multiplicative inverse. `A2S.NQ.B.5.B`

Algebra and Functions

A. Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible. A2S.AF.A

  6. Factor polynomials using common factoring techniques, and use the factored form of a polynomial to reveal the zeros of the function it defines. A2S.AF.A.6

  7. Prove polynomial identities and use them to describe numerical relationships. A2S.AF.A.7

B. Finding solutions to an equation, inequality, or system of equations or inequalities requires the checking of candidate solutions, whether generated analytically or graphically, to ensure that solutions are found and that those found are not extraneous. A2S.AF.B

  8. Explain why extraneous solutions to an equation may arise and how to check to be sure that a candidate solution satisfies an equation. Extend to radical equations. A2S.AF.B.8

C. The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution. A2S.AF.C

  9. For exponential models, express as a logarithm the solution to ab^{ct} = d, where a, c, and d are real numbers and the base b is 2 or 10; evaluate the logarithm using technology to solve an exponential equation. A2S.AF.C.9

D. Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts—in particular, contexts that arise in relation to linear, quadratic, and exponential situations. A2S.AF.D

  10. Create equations and inequalities in one variable and use them to solve problems. Extend to equations arising from polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions. A2S.AF.D.10

  11. Solve quadratic equations with real coefficients that have complex solutions. A2S.AF.D.11

  12. Solve simple equations involving exponential, radical, logarithmic, and trigonometric functions using inverse functions. A2S.AF.D.12

  13. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales and use them to make predictions. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions. A2S.AF.D.13

E. Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities—including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology). A2S.AF.E

14. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x). A2S.AF.E.14

    a. Find the approximate solutions of an equation graphically, using tables of values, or finding successive approximations, using technology where appropriate. Extend to cases where f(x) and/or g(x) are polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions. A2S.AF.E.14.A

F. Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g., f(x) = x^2), recursive definitions, tables, and graphs. A2S.AF.F

15. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend to polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions. A2S.AF.F.15

G. Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family. A2S.AF.G

16. Identify the effect on the graph of replacing f(x) by f(x) + k, k·f(x), f(k·x), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions. A2S.AF.G.16

H. Functions can be represented graphically, and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation. A2S.AF.H

17. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions. A2S.AF.H.17

18. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions. A2S.AF.H.18

19. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions. A2S.AF.H.19

20. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Extend

to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions. A2S.AF.H.20

a. Graph polynomial functions expressed symbolically, identifying zeros when suitable factorizations are available, and showing end behavior. A2S.AF.H.20.A

b. Graph sine and cosine functions expressed symbolically, showing period, midline, and amplitude. A2S.AF.H.20.B

c. Graph logarithmic functions expressed symbolically, showing intercepts and end behavior. A2S.AF.H.20.C

d. Graph reciprocal functions expressed symbolically, identifying horizontal and vertical asymptotes. A2S.AF.H.20.D

e. Graph square root and cube root functions expressed symbolically. A2S.AF.H.20.E

f. Compare the graphs of inverse functions and the relationships between their key features, including but not limited to quadratic, square root, exponential, and logarithmic functions. A2S.AF.H.20.F

21. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle, building on work with non-right triangle trigonometry. A2S.AF.H.21

I. Functions model a wide variety of real situations and can help students understand the processes of making and changing assumptions, assigning variables, and finding solutions to contextual problems. A2S.AF.I

22. Use the mathematical modeling cycle to solve real-world problems involving polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions, from the simplification of the problem through the solving of the simplified problem, the interpretation of its solution, and the checking of the solution's feasibility. A2S.AF.I.22

Data Analysis, Statistics, and Probability

A. Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks. A2S.DSP.A

   23. Use mathematical and statistical reasoning about normal distributions to draw conclusions and assess risk; limit to informal arguments. A2S.DSP.A.23

B. Making and defending informed data-based decisions is a characteristic of a quantitatively literate person. A2S.DSP.B

   24. Design and carry out an experiment or survey to answer a question of interest, and write an informal persuasive argument based on the results. A2S.DSP.B.24

C. Distributions of quantitative data (continuous or discrete) in one variable should be described in the context of the data with respect to what is typical (the shape, with appropriate measures of center and variability, including standard deviation) and what is not (outliers), and these characteristics can be used to compare two or more subgroups with respect to a variable. A2S.DSP.C

   25. From a normal distribution, use technology to find the mean and standard deviation and estimate population percentages by applying the empirical rule. A2S.DSP.C.25

      a. Use technology to determine if a given set of data is normal by applying the empirical rule. A2S.DSP.C.25.A

      b. Estimate areas under a normal curve to solve problems in context, using calculators, spreadsheets, and tables as appropriate. A2S.DSP.C.25.B

D. Study designs are of three main types: sample survey, experiment, and observational study. A2S.DSP.D

   26. Describe the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. A2S.DSP.D.26

E. The role of randomization is different in randomly selecting samples and in randomly assigning subjects to experimental treatment groups. A2S.DSP.E

   27. Distinguish between a statistic and a parameter and use statistical processes to make inferences about population parameters based on statistics from random samples from that population. A2S.DSP.E.27

   28. Describe differences between randomly selecting samples and randomly assigning subjects to experimental treatment groups in terms of inferences drawn regarding a population versus regarding cause and effect. A2S.DSP.E.28

F. The scope and validity of statistical inferences are dependent on the role of randomization in the study design. A2S.DSP.F

   29. Explain the consequences, due to uncontrolled variables, of non-randomized assignment of subjects to groups in experiments. A2S.DSP.F.29

G. Bias, such as sampling, response, or nonresponse bias, may occur in surveys, yielding results that are not representative of the population of interest. A2S.DSP.G

30. Evaluate where bias, including sampling, response, or nonresponse bias, may occur in surveys, and whether results are representative of the population of interest. A2S.DSP.G.30

H. The larger the sample size, the less the expected variability in the sampling distribution of a sample statistic. A2S.DSP.H

31. Evaluate the effect of sample size on the expected variability in the sampling distribution of a sample statistic. A2S.DSP.H.31

   a. Simulate a sampling distribution of sample means from a population with a known distribution, observing the effect of the sample size on the variability. A2S.DSP.H.31.A

   b. Demonstrate that the standard deviation of each simulated sampling distribution is the known standard deviation of the population divided by the square root of the sample size. A2S.DSP.H.31.B

I. The sampling distribution of a sample statistic formed from repeated samples for a given sample size drawn from a population can be used to identify typical behavior for that statistic. Examining several such sampling distributions leads to estimating a set of plausible values for the population parameter, using the margin of error as a measure that describes the sampling variability. A2S.DSP.I

32. Produce a sampling distribution by repeatedly selecting samples of the same size from a given population or from a population simulated by bootstrapping (resampling with replacement from an observed sample). Do initial examples by hand, then use technology to generate a large number of samples. A2S.DSP.I.32

   a. Verify that a sampling distribution is centered at the population mean and approximately normal if the sample size is large enough. A2S.DSP.I.32.A

   b. Verify that 95% of sample means are within two standard deviations of the sampling distribution from the population mean. A2S.DSP.I.32.B

   c. Create and interpret a 95% confidence interval based on an observed mean from a sampling distribution. A2S.DSP.I.32.C

33. Use data from a randomized experiment to compare two treatments; limit to informal use of simulations to decide if an observed difference in the responses of the two treatment groups is unlikely to have occurred due to randomization alone, thus implying that the difference between the treatment groups is meaningful. A2S.DSP.I.33

Geometry and Measurement

A. When an object is the image of a known object under a similarity transformation, a length, area, or volume on the image can be computed by using proportional relationships. A2S.GM.A

  34. Define the radian measure of an angle as the constant of proportionality of the length of an arc it intercepts to the radius of the circle; in particular, it is the length of the arc intercepted on the unit circle. A2S.GM.A.34

B. Recognizing congruence, similarity, symmetry, measurement opportunities, and other geometric ideas, including right triangle trigonometry in real-world contexts, provides a means of building understanding of these concepts and is a powerful tool for solving problems related to the physical world in which we live. A2S.GM.B

  35. Choose trigonometric functions (sine and cosine) to model periodic phenomena with specified amplitude, frequency, and midline. A2S.GM.B.35
  36. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios. A2S.GM.B.36
  37. Derive and apply the formula $A = ½·ab·\sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side, extending the domain of sine to include right and obtuse angles. A2S.GM.B.37
  38. Derive and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles. Extend the domain of sine and cosine to include right and obtuse angles. A2S.GM.B.38

Mathematical Modeling

Modeling

A. Mathematical modeling and statistical problem-solving are extensive, cyclical processes that can be used to answer significant real-world problems. MM.M.A

  1. Use the full Mathematical Modeling Cycle or Statistical Problem-Solving Cycle to answer a real-world problem of particular student interest, incorporating standards from across the course. MM.M.A.1

Financial Planning and Management

A. Mathematical models involving growth and decay are useful in solving real-world problems involving borrowing and investing; spreadsheets are a frequently-used and powerful tool to assist with modeling financial situations. MM.FPM.A

2. Use elements of the Mathematical Modeling Cycle to solve real-world problems involving finances. MM.FPM.A.2

3. Organize and display financial information using arithmetic sequences to represent simple interest and straight-line depreciation. MM.FPM.A.3

4. Organize and display financial information using geometric sequences to represent compound interest and proportional depreciation, including periodic (yearly, monthly, weekly) and continuous compounding. MM.FPM.A.4

   a. Explain the relationship between annual percentage yield (APY) and annual percentage rate (APR) as values for r in the formulas A=P(1+r)t and A=Pert. MM.FPM.A.4.A

5. Compare simple and compound interest, and straight-line and proportional depreciation. MM.FPM.A.5

6. Investigate growth and reduction of credit card debt using spreadsheets, including variables such as beginning balance, payment structures, credits, interest rates, new purchases, finance charges, and fees. MM.FPM.A.6

7. Compare and contrast housing finance options including renting, leasing to purchase, purchasing with a mortgage, and purchasing with cash. MM.FPM.A.7

   a. Research and evaluate various mortgage products available to consumers. MM.FPM.A.7.A

   b. Compare monthly mortgage payments for different terms, interest rates, and down payments. MM.FPM.A.7.B

   c. Analyze the financial consequence of buying a home (mortgage payments vs. potentially increasing resale value) versus investing the money saved when renting, assuming that renting is the less expensive option. MM.FPM.A.7.C

   8. Investigate the advantages and disadvantages of various means of paying for an automobile, including leasing, purchasing by cash, and purchasing by loan. MM.FPM.A.8

Design in Three Dimensions

A. Two- and three-dimensional representations, coordinates systems, geometric transformations, and scale models are useful tools in planning, designing, and constructing solutions to real-world problems. MM.D3D.A

9. Use the Mathematical Modeling Cycle to solve real-world problems involving the design of three-dimensional objects. MM.D3D.A.9

10. Construct a two-dimensional visual representation of a three-dimensional object or structure. MM.D3D.A.10

   a. Determine the level of precision and the appropriate tools for taking the measurements in constructing a two-dimensional visual representation of a three-dimensional object or structure. MM.D3D.A.10.A

   b. Create an elevation drawing to represent a given solid structure, using technology where appropriate. MM.D3D.A.10.B

   c. Determine which measurements cannot be taken directly and must be calculated based on other measurements when constructing a two-dimensional visual representation of a three-dimensional object or structure. MM.D3D.A.10.C

   d. Determine an appropriate means to visually represent an object or structure, such as drawings on paper or graphics on computer screens. MM.D3D.A.10.D

11. Plot coordinates on a three-dimensional Cartesian coordinate system and use relationships between coordinates to solve design problems. MM.D3D.A.11

   a. Describe the features of a three-dimensional Cartesian coordinate system and use them to graph points. MM.D3D.A.11.A

   b. Graph a point in space as the vertex of a right prism drawn in the appropriate octant with edges along the x, y, and z axes. MM.D3D.A.11.B

   c. Find the distance between two objects in space given the coordinates of each. MM.D3D.A.11.C

   d. Find the midpoint between two objects in space given the coordinates of each. MM.D3D.A.11.D

12. Use technology and other tools to explore the results of simple transformations using three-dimensional coordinates, including translations in the x, y, and/or z directions; rotations of 90°, 180°, or 270° about the x, y, and z axes; reflections over the xy, yz, and xy planes; and dilations from the origin. MM.D3D.A.12

13. Create a scale model of a complex three-dimensional structure based on observed measurements and indirect measurements, using translations, reflections, rotations, and dilations of its components. MM.D3D.A.13

Creating Functions to Model Change in the Environment and Society

B. Functions can be used to represent general trends in conditions that change over time and to predict future conditions based on present observations. MM.D3D.B

14. Use elements of the Mathematical Modeling Cycle to make predictions based on measurements that change over time, including motion, growth, decay, and cycling. MM.D3D.B.14

15. Use regression with statistical graphing technology to determine an equation that best fits a set of bivariate data, including nonlinear patterns. MM.D3D.B.15

    a. Create a scatter plot with a sufficient number of data points to predict a pattern. MM.D3D.B.15.A

    b. Describe the overall relationship between two quantitative variables (increase, decrease, linearity, concavity, extrema, inflection) or pattern of change. MM.D3D.B.15.B

    c. Make a prediction based upon patterns. MM.D3D.B.15.C

16. Create a linear representation of non-linear data and interpret solutions, using technology and the process of linearization with logarithms. MM.D3D.B.16

Modeling to Interpret Statistical Studies

C. Statistical studies allow a conclusion to be drawn about a population that is too large to survey completely or about cause and effect in an experiment. MM.D3D.C

17. Use the Statistical Problem Solving Cycle to answer real-world questions. MM.D3D.C.17

18. Construct a probability distribution based on empirical observations of a variable. MM.D3D.C.18

    a. Estimate the probability of each value for a random variable based on empirical observations or simulations, using technology. MM.D3D.C.18.A

    b. Represent a probability distribution by a relative frequency histogram and/or a cumulative relative frequency graph. MM.D3D.C.18.B

    c. Find the mean, standard deviation, median, and interquartile range of a probability distribution and make long-term predictions about future possibilities. Determine which measures are most appropriate based upon the shape of the distribution. MM.D3D.C.18.C

19. Construct a sampling distribution for a random event or random sample. MM.D3D.C.19

    a. Use the binomial theorem to construct the sampling distribution for the number of successes in a binary event or the number of positive responses to a yes/no question in a random sample. MM.D3D.C.19.A

    b. Use the normal approximation of a proportion from a random event or sample when conditions are met. MM.D3D.C.19.B

    c. Use the central limit theorem to construct a normal sampling distribution for the sample mean when conditions are met. MM.D3D.C.19.C

    d. Find the long-term probability of a given range of outcomes from a random event or random sample. MM.D3D.C.19.D

20. Perform inference procedures based on the results of samples and experiments. MM.D3D.C.20

    a. Use a point estimator and margin of error to construct a confidence interval for a proportion or mean. MM.D3D.C.20.A

    b. Interpret a confidence interval in context and use it to make strategic decisions. MM.D3D.C.20.B

    c. Perform a significance test for null and alternative hypotheses. MM.D3D.C.20.C

    d. Interpret the significance level of a test in the context of error probabilities, and use the results to make strategic decisions. MM.D3D.C.20.D

21. Critique the validity of reported conclusions from statistical studies in terms of bias and random error probabilities. MM.D3D.C.21

22. Conduct a randomized study on a topic of student interest (sample or experiment) and draw conclusions based upon the results. MM.D3D.C.22

Applications of Finite Mathematics

Logical Reasoning

A. The validity of a statement or argument can be determined using the models and language of first order logic. FM.LR.A

1. Represent logic statements in words, with symbols, and in truth tables, including conditional, biconditional, converse, inverse, contrapositive, and quantified statements. FM.LR.A.1
2. Represent logic operations such as and, or, not, nor, and x or (exclusive or) in words, with symbols, and in truth tables. FM.LR.A.2
3. Use truth tables to solve application-based logic problems and determine the truth value of simple and compound statements including negations and implications. FM.LR.A.3
   a. Determine whether statements are equivalent and construct equivalent statements. FM.LR.A.3.A
4. Determine whether a logical argument is valid or invalid, using laws of logic such as the law of syllogism and the law of detachment. FM.LR.A.4
   a. Determine whether a logical argument is a tautology or a contradiction. FM.LR.A.4.A
5. Prove a statement indirectly by proving the contrapositive of the statement. FM.LR.A.5

Advanced Counting

A. Complex counting problems can be solved efficiently using a variety of techniques. FM.AC.A

6. Use multiple representations and methods for counting objects and developing more efficient counting techniques. Note: Representations and methods may include tree diagrams, lists, manipulatives, overcounting methods, recursive patterns, and explicit formulas. FM.AC.A.6

7. Develop and use the Fundamental Counting Principle for counting independent and dependent events. FM.AC.A.7

    a. Use various counting models (including tree diagrams and lists) to identify the distinguishing factors of a context in which the Fundamental Counting Principle can be applied. FM.AC.A.7.A

8. Using application-based problems, develop formulas for permutations, combinations, and combinations with repetition and compare student-derived formulas to standard representations of the formulas. FM.AC.A.8

    a. Identify differences between applications of combinations and permutations. FM.AC.A.8.A

    b. Using application-based problems, calculate the number of permutations of a set with n elements. Calculate the number of permutations of r elements taken from a set of n elements. FM.AC.A.8.B

    c. Using application-based problems, calculate the number of subsets of size r that can be chosen from a set of n elements, explaining this number as the number of combinations "n choose r." FM.AC.A.8.C

    d. Using application-based problems, calculate the number of combinations with repetitions of r elements from a set of n elements as "(n + r – 1) choose r." FM.AC.A.8.D

9. Use various counting techniques to determine probabilities of events. FM.AC.A.9

10. Use the Pigeonhole Principle to solve counting problems. FM.AC.A.10

Recursion

A. Recursion is a method of problem solving where a given relation or routine operation is repeatedly applied. FM.R.A

11. Find patterns in application problems involving series and sequences, and develop recursive and explicit formulas as models to understand and describe sequential change. FM.R.A.11
12. Determine characteristics of sequences, including the Fibonacci Sequence, the triangular numbers, and pentagonal numbers. FM.R.A.12
13. Use the recursive process and difference equations to create fractals, population growth models, sequences, and series. FM.R.A.13
14. Use mathematical induction to prove statements involving the positive integers. FM.R.A.14
15. Develop and apply connections between Pascal's Triangle and combinations. FM.R.A.15

Networks

A. Complex problems can be modeled using vertex and edge graphs and characteristics of the different structures are used to find solutions. FM.N.A

16. Use vertex and edge graphs to model mathematical situations involving networks. FM.N.A.16
    a. Identify properties of simple graphs, complete graphs, bipartite graphs, complete bipartite graphs, and trees. FM.N.A.16.A
17. Solve problems involving networks through investigation and application of existence and nonexistence of Euler paths, Euler circuits, Hamilton paths, and Hamilton circuits. FM.N.A.17
    a. Develop optimal solutions of application-based problems using existing and student-created algorithms. FM.N.A.17.A
    b. Give an argument for graph properties. FM.N.A.17.B
18. Apply algorithms relating to minimum weight spanning trees, networks, flows, and Steiner trees. FM.N.A.18
    a. Use shortest path techniques to find optimal shipping routes. FM.N.A.18.A
    b. Show that every connected graph has a minimal spanning tree. FM.N.A.18.B
    c. Use Kruskal's Algorithm and Prim's Algorithm to determine the minimal spanning tree of a weighted graph. FM.N.A.18.C
19. Use vertex-coloring, edge-coloring, and matching techniques to solve application-based problems involving conflict. FM.N.A.19
20. Determine the minimum time to complete a project using algorithms to schedule tasks in order, including critical path analysis, the list-processing algorithm, and student-created algorithms. FM.N.A.20
21. Use the adjacency matrix of a graph to determine the number of walks of length n in a graph. FM.N.A.21

Fairness and Democracy

A. Various methods for determining a winner in a voting system can result in paradoxes or other issues of fairness. FM.FD.A

22. Analyze advantages and disadvantages of different types of ballot voting systems. FM.FD.A.22
    a. Identify impacts of using a preferential ballot voting system and compare it to single candidate voting and other voting systems. FM.FD.A.22.A
    b. Analyze the impact of legal and cultural features of political systems on the mathematical aspects of elections. FM.FD.A.22.B
23. Apply a variety of methods for determining a winner using a preferential ballot voting system, including plurality, majority, run-off with majority, sequential run-off with majority, Borda count, pairwise comparison, Condorcet, and approval voting. FM.FD.A.23
24. Identify issues of fairness for different methods of determining a winner using a preferential voting ballot and other voting systems and identify paradoxes that can result. FM.FD.A.24
25. Use methods of weighted voting and identify issues of fairness related to weighted voting. Example: determine the power of voting bodies using the Banzhaf power index FM.FD.A.25
    a. Distinguish between weight and power in voting. FM.FD.A.25.A

Fair Division

A. Methods used to solve non-trivial problems of division of objects often reveal issues of fairness. FM.FDV.A

26. Explain and apply mathematical aspects of fair division, with respect to classic problems of apportionment, cake cutting, and estate division. Include applications in other contexts and modern situations. FM.FDV.A.26
27. Identify and apply historic methods of apportionment for voting districts including Hamilton, Jefferson, Adams, Webster, and Huntington-Hill. Identify issues of fairness and paradoxes that may result from methods. FM.FDV.A.27
28. Use spreadsheets to examine apportionment methods in large problems. FM.FDV.A.28

Information Processing

A. Effective systems for sending and receiving information include components that impact accuracy, efficiency, and security. `FM.IP.A`

   29. Critically analyze issues related to information processing including accuracy, efficiency, and security. `FM.IP.A.29`

   30. Apply ciphers (encryption and decryption algorithms) and cryptosystems for encrypting and decrypting including symmetric-key or public-key systems. `FM.IP.A.30`

      a. Use modular arithmetic to apply RSA (Rivest-Shamir-Adleman) public-key cryptosystems. `FM.IP.A.30.A`

      b. Use matrices and their inverses to encode and decode messages. `FM.IP.A.30.B`

   31. Apply error-detecting codes and error-correcting codes to determine accuracy of information processing. `FM.IP.A.31`

   32. Apply methods of data compression. `FM.IP.A.32`

Precalculus

Number and Quantity

A. Perform arithmetic operations with complex numbers. PC.NQ.A

1. Define the constant e in a variety of contexts. PC.NQ.A.1
    - a. Explore the behavior of the function y = e^x and its applications. PC.NQ.A.1.A
    - b. Explore the behavior of ln(x), the logarithmic function with base e, and its applications. PC.NQ.A.1.B
2. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. PC.NQ.A.2

B. Represent complex numbers and their operations on the complex plane. PC.NQ.B

3. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. PC.NQ.B.3
4. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. PC.NQ.B.4
5. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. PC.NQ.B.5

C. Use complex numbers in polynomial identities and equations. PC.NQ.C

6. Analyze possible zeros for a polynomial function over the complex numbers by applying the Fundamental Theorem of Algebra, using a graph of the function, or factoring with algebraic identities. PC.NQ.C.6

D. Understand limits of functions. PC.NQ.D

7. Determine numerically, algebraically, and graphically the limits of functions at specific values and at infinity. PC.NQ.D.7
    - a. Apply limits of functions at specific values and at infinity in problems involving convergence and divergence. PC.NQ.D.7.A

E. Represent and model with vector quantities. PC.NQ.E

8. Explain that vector quantities have both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes. PC.NQ.E.8
9. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. PC.NQ.E.9
10. Solve problems involving velocity and other quantities that can be represented by vectors. PC.NQ.E.10
11. Find the scalar (dot) product of two vectors as the sum of the products of corresponding components and explain its relationship to the cosine of the angle formed by two vectors. PC.NQ.E.11

F. Perform operations on vectors. PC.NQ.F

12. Add and subtract vectors. PC.NQ.F.12

    a. Add vectors end-to-end, component-wise, and by the parallelogram rule, understanding that the magnitude of a sum of two vectors is not always the sum of the magnitudes. PC.NQ.F.12.A

    b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. PC.NQ.F.12.B

    c. Explain vector subtraction, v – w, as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. PC.NQ.F.12.C

13. Multiply a vector by a scalar. PC.NQ.F.13

    a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. PC.NQ.F.13.A

    b. Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when |c|v ≠ 0, the direction of cv is either along v (for c > 0) or against v (for c < 0). PC.NQ.F.13.B

14. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. PC.NQ.F.14

Algebra

A. Write expressions in equivalent forms to solve problems. PC.A.A

15. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems, extending to infinite geometric series. PC.A.A.15

B. Understand the relationship between zeros and factors of polynomials. PC.A.B

16. Derive and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). PC.A.B.16

C. Use polynomial identities to solve problems. PC.A.C

17. Know and apply the Binomial Theorem for the expansion of (x + y)ⁿ in powers of x and y for a positive integer, n, where x and y are any numbers. PC.A.C.17

D. Rewrite rational expressions. PC.A.D

18. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated cases, a computer algebra system. PC.A.D.18

19. Add, subtract, multiply, and divide rational expressions. PC.A.D.19
    a. Explain why rational expressions form a system analogous to the rational numbers, which is closed under addition, subtraction, multiplication, and division by a non-zero rational expression. PC.A.D.19.A

E. Understand solving equations as a process of reasoning and explain the reasoning. PC.A.E

20. Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a clear-cut solution. Construct a viable argument to justify a solution method. Include equations that may involve linear, quadratic, polynomial, exponential, logarithmic, absolute value, radical, rational, piecewise, and trigonometric functions, and their inverses. PC.A.E.20

21. Solve simple rational equations in one variable, and give examples showing how extraneous solutions may arise. PC.A.E.21

F. Solve systems of equations. PC.A.F

22. Represent a system of linear equations as a single matrix equation in a vector variable. PC.A.F.22

23. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater). PC.A.F.23

Functions

A. Interpret functions that arise in applications in terms of the context. PC.F.A

24. Compare and contrast families of functions and their representations algebraically, graphically, numerically, and verbally in terms of their key features. Families of functions include but are not limited to linear, quadratic, polynomial, exponential, logarithmic, absolute value, radical, rational, piecewise, trigonometric, and their inverses. PC.F.A.24

25. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Extend from polynomial, exponential, logarithmic, and radical to rational and all trigonometric functions. PC.F.A.25

    a. Find the difference quotient f(x + △x) - f(x)/△x of a function and use it to evaluate the average rate of change at a point. PC.F.A.25.A

    b. Explore how the average rate of change of a function over an interval (presented symbolically or as a table) can be used to approximate the instantaneous rate of change at a point as the interval decreases. PC.F.A.25.B

B. Analyze functions using different representations. PC.F.B

26. Graph functions expressed symbolically and show key features of the graph, by hand and using technology. Use the equation of functions to identify key features in order to generate a graph. PC.F.B.26

    a. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. PC.F.B.26.A

    b. Graph trigonometric functions and their inverses, showing period, midline, amplitude, and phase shift. PC.F.B.26.B

C. Build a function that models a relationship between two quantities. PC.F.C

27. Compose functions. Extend to polynomial, trigonometric, radical, and rational functions. PC.F.C.27

D. Build new functions from existing functions. PC.F.D

28. Find inverse functions. PC.F.D.28

    a. Given that a function has an inverse, write an expression for the inverse of the function. PC.F.D.28.A

    b. Verify by composition that one function is the inverse of another. PC.F.D.28.B

    c. Read values of an inverse function from a graph or a table, given that the function has an inverse. PC.F.D.28.C

    d. Produce an invertible function from a non-invertible function by restricting the domain. PC.F.D.28.D

29. Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents. Extend from logarithms with base 2 and 10 to a base of e. PC.F.D.29

30. Identify the effect on the graph of replacing f(x) by f(x) + k, k·f(x), f(k·x), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Extend the analysis to include all trigonometric, rational, and general piecewise-defined functions with and without technology. PC.F.D.30

31. Graph conic sections from second-degree equations, extending from circles and parabolas to ellipses and hyperbolas, using technology to discover patterns. PC.F.D.31

    a. Graph conic sections given their standard form. PC.F.D.31.A

    b. Identify the conic section that will be formed, given its equation in general form. PC.F.D.31.B

E. Recognize attributes of trigonometric functions and solve problems involving trigonometry. PC.F.E

32. Solve application-based problems involving parametric and polar equations. PC.F.E.32

    a. Graph parametric and polar equations. PC.F.E.32.A

    b. Convert parametric and polar equations to rectangular form. PC.F.E.32.B

F. Extend the domain of trigonometric functions using the unit circle. PC.F.F

33. Use special triangles to determine geometrically the values of sine, cosine, and tangent for π/3, π/4, and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π -x, π + x, and 2π - x in terms of their values for x, where x is any real number. PC.F.F.33

34. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. PC.F.F.34

G. Model periodic phenomena with trigonometric functions. PC.F.G

35. Demonstrate that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. PC.F.G.35

36. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. PC.F.G.36

H. Prove and apply trigonometric identities. PC.F.H

37. Use trigonometric identities to solve problems. PC.F.H.37

    a. Use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ to derive the other forms of the identity. PC.F.H.37.A

    b. Use the angle sum formulas for sine, cosine, and tangent to derive the double angle formulas. PC.F.H.37.B

    c. Use the Pythagorean and double angle identities to prove other simple identities. PC.F.H.37.C